1. For the following functions, algebraically determine the points of discontinuity of f. Justify your answer.

   (a) $f(x) = \begin{cases} x+10 & \text{if } x \le -2 \\ x^2+4 & \text{if } -2 < x < 6 \\ 3x+2 & \text{if } 6 \le x \end{cases}$.

   (b) $f(x) = \begin{cases} |x| & \text{if } x < 7 \\ x^2+2x+1 & \text{if } 7 \le x \le 10 \\ 3x^2-8x & \text{if } 10 < x \end{cases}$.

2. Find the value of k so that $f(x) = \begin{cases} 3x^2+2x+1 & \text{if } x \le 1 \\ kx+3 & \text{if } x > 1 \end{cases}$ is continuous on $\mathbb{R}$.

3. For each of the following graphs of a function $f(x)$, answer the following questions.

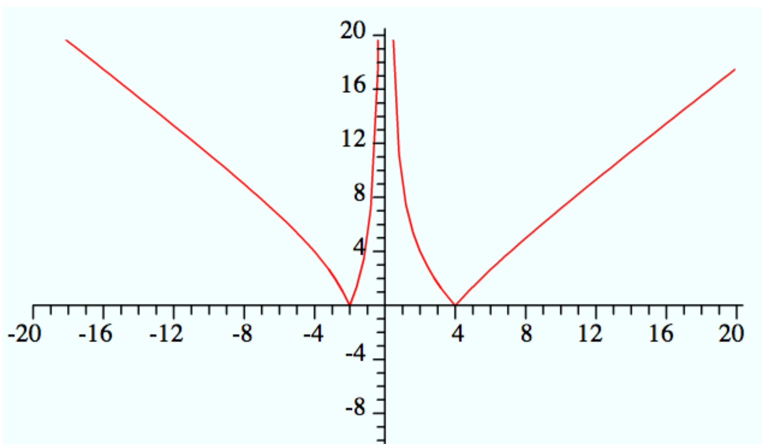
   (a) Find the values of x where f is NOT differentiable and why.

   (b) Find the values of x for which f' is positive.

   (c) Find the values of x for which f' is negative.

   (d) Find the values of x for which f' is zero.

(i)



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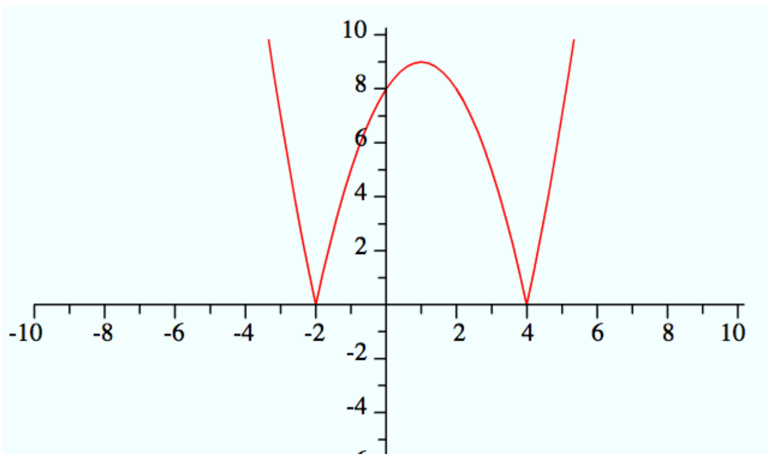
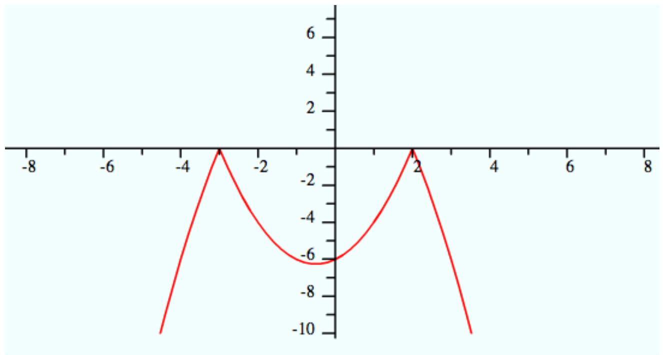


(iii) 

4. Use the limit definition of the derivative to find $f'(x)$ for the following functions.
   (a) $f(x) = \sqrt{x+5}$
   (b) $f(x) = \dfrac{3x+1}{2x+4}$
   (c) $f(x) = x + \dfrac{1}{x}$
   (d) $f(x) = x^2 + 5x$

5. For the the following limits, find a $\delta > 0$ such that if $|x - a| < \delta$, then $|f(x) - L| < \varepsilon$. Show all work used to arrive at your answer
   (a) $\lim_{x\to 3}(2x+5) = 11$, and $\varepsilon = .04$
   (b) $\lim_{x\to -7}\sqrt{9-x} = 4$ and $\varepsilon = .023$
   (c) $\lim_{x\to 4}(3x+9) = 21$ and $\varepsilon = .017$

6. A particle travels in a straight line according to the position function $s(t) = 9t^2 + 5t$ meters where t is in seconds.
   (a) Find the velocity as a function of t. Include the units.
   (b) Find the acceleration as a function of t. Include the units.
   (c) What is the acceleration when the velocity is 0? Include the units.

7. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in meters) after t seconds is given by $H = 10t - 1.76t^2$.
   (a) Find the velocity of the rock after one second.
   (b) Find the velocity of the rock when t = a.
   (c) When will the rock hit the surface?
   (d) With what velocity will the rock hit the surface?

8. Consider a spring which is attached to a wall on the left and has a mass attached on the right. The equilibrium position when the spring is still corresponds to $x = 0$ and the positive direction is to the right. If the mass is pulled out to the right, the mass vibrates horizontally on a smooth level surface according to the equation of motion $x(t) = 2\cos t + 7\sin t$ where x is the displacement from the equilibrium position.

(a) Find the velocity, v(t), and acceleration, a(t), functions and graph them on the interval $[0, 2\pi]$.

(b) What is the initial position, velocity and acceleration?

(c) At what times is the speed (which equals $|v(t)|$) largest and what is the largest speed?

9. Compute the following limits algebraically using limit laws. If the limit is $+\infty$ or $-\infty$, then say so. You must show work or justification for each of your answers.

(a) $\displaystyle\lim_{x\to\infty} \frac{3e^x + 1}{4e^x - 1}$

(b) $\displaystyle\lim_{x\to\infty} \frac{3e^{2x} + 1}{4e^x - 1}$

(c) $\displaystyle\lim_{t\to 0} \frac{\sin 4t}{\sin 9t}$

(d) $\displaystyle\lim_{x\to -\infty} \frac{x^2 + 3x}{\sqrt{3x^4 + 1}}$

(e) $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{3x}$

(f) $\displaystyle\lim_{x\to\infty} \frac{2x^2 + 3x}{4x^3 + 9x + 1}$

10. For the following functions determine the vertical and horizontal asymptotes, if any exist.

(a) $f(x) = \dfrac{\sqrt{9x^4 - x}}{x^2 - 1}$

(b) $f(x) = \dfrac{x^3 - 2x^2 + x}{x^2 - 3x + 2}$

(c) $f(x) = \dfrac{1 + e^{4x}}{1 - 3e^{4x}}$

11. Use differentiation rules to find the derivatives of each of the following functions. Show your work. There is no need to simplify your answers beyond easy simplifications.

(a) $f(x) = \sqrt[3]{x^3 + 1}$

(b) $f(t) = \dfrac{6t^4 - 2t + 1}{\cos(t^2 + 1)}$

(c) $g(x) = \sin e^{2x}$

(d) $h(z) = z^3 + 9z + 4 + \dfrac{3}{z^3}$

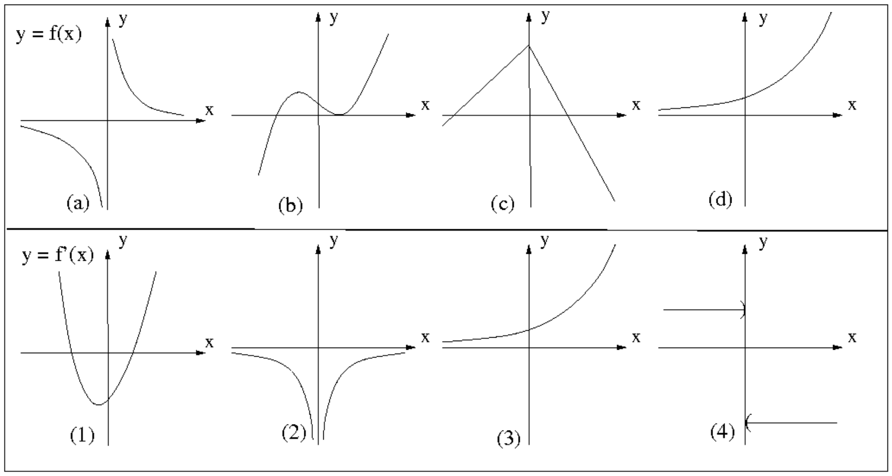
(e) $f(x) = \tan^2(x^3)$

12. If $f(t) = te^{2t}$, find $f'(t)$ and $f''(t)$.

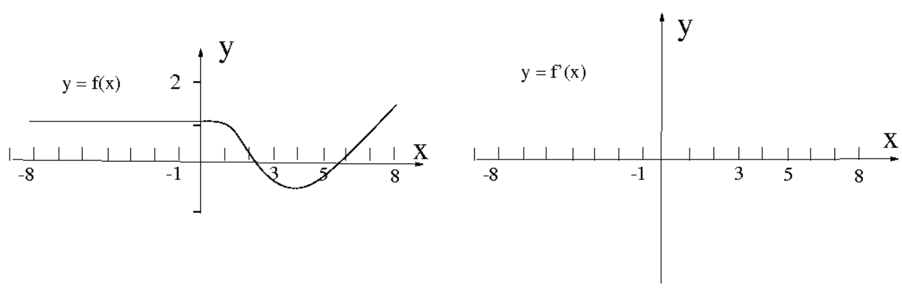
13. If $s(x) = x^2 + 4x + 9$, find $s'(x)$ and $s''(x)$.

14. If $f(t) = \dfrac{3t + 1}{4t + 1}$, find $f'(t)$ and $f''(t)$.

15. Match the graph of each function in (a)-(d) with the graph of its derivative in (1)-(4). Give reason for your choice.



16. Given the sketch of $y = f(x)$, sketch a graph of $f'(x)$.



17. Find the equation of the tangent line to the curve $y = x^3 + 3x + 1$ at $x = 0$.

18. Find the equation of the tangent line to the curve $y = \sqrt{2x^2 + 3x + 1}$ at $x = 3$.

19. Find the equation of the tangent line to the curve $y = \dfrac{x^2 + 1}{x^2 - 1}$ at $x = 0$.