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This is an example of an *Equiprobable Probability Space*.

Terminology

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A subset of the sample space is referred to as an *event*. We'll spend time on events later on.

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In our analyses, let's assume every bet is for $1.

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If he (or she) plays 38 times, he could expect to win about one time, coming out \$35 ahead that time, but losing about 37 times, coming out \$1 behind those times, so after playing 38 times he can expect to be about \$2 behind. That works out to losing about $\$\frac{2}{38} = \$\frac{1}{19}$ per bet.

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If we denote the probability X takes on a certain value k by $P(X = k)$, then $P(X = 35) = \frac{1}{38}$ and $P(X = -1) = \frac{37}{38}$.

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In this case, we expect $E(X) = -\frac{1}{19}$, since we expect to lose an average of $\$\frac{1}{19}$ per bet.

Calculating Mathematical Expectation

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Abstracting to Get a Formula

The calculation $E(X) = 35P(X = 35) + (-1)P(X = -1)$ suggests we calculate mathematical expectation by taking each possible value of the random variable, multiply it by the probability the random variable takes on that value, and add the products together.

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Symbolically, we may write this as

$E(X) = \sum kP(X = k)$,
where the sum is taken over all the possible values for X.

Probability Spaces

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A probability space is a sample space $\mathcal{S}$ for which a probability $p(x)$ has been assigned to each sample point x such that the sum $\sum_{x \in \mathcal{S}} p(x)$ of the probabilities is 1.

Examples

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The first example is known as an *Equiprobable Space*; the second is not an equiprobable space.

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