UP TO EQUIMORPHISM, HYPERARITHMETIC IS COMPUTABLE.

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ABSTRACT. Clifford Spector proved in 1955 that every hyperarithmetic (or equivalently Δ^1_1) well ordering is isomorphic to a computable one. The direct generalization of Spector's Theorem to the class of linear orderings does not hold; It is not the case that every linear ordering with a hyperarithmetic presentation is isomorphic to a computable one. It is known that for every non-computable Turing degree, there is a linear ordering of that degree which has no computable copy. This was proved by Knight extending a result of Downey. But there are other ways in which we can generalize Spector's Theorem. We say that two linear orderings are *equimorphic* if each one can be embedded into the other one. Observe that if a linear ordering and a well ordering are equimorphic, then they are actually isomorphic. We prove the following generalization of Spector's theorem: every hyperarithmetic linear ordering is equimorphic to a computable one. Moreover, we prove that every linear ordering of Hausdorff rank less than ω_1^{CK} is equimorphic to a computable one.

The equimorphism relation on linear orderings is very natural. Many properties of linear orderings are invariant under equimorphisms, as, for example, extendibility, indecomposability, well foundedness, being scattered, or having a certain Hausdorff rank. The structure of equimorphism types of countable linear orderings ordered by embeddablity is also an interesting object. For example, Fraïssé's conjecture (which was proved by Laver) is the statement that says that this partial has no infinite descending sequences and no infinite antichains. This statement has interested logicians because of its proof theoretic complexity. We prove that for every ordinal $\alpha < \omega_1^{CK}$, the partial ordering of equimorphism types of linear orderings of Hausdorff rank less than α ordered by embeddablity, is computably presentable.

In this talk, I am planning describe all these results. Also, I will give and outline of the proofs and show some key ideas.

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