References for Analysis on Noncompact Manifolds

Much of the analysis one needs to know to study problems on noncompact manifolds is spread throughout the literature. However, here are some good starting points.

- *The Atiyah-Patodi-Singer Index Theorem* by Richard Melrose. This is ostensibly a book about an index theorem for manifolds with boundary, but as Melrose states in the introduction, it's really an excuse to preach about the virtues of his b-calculus. The idea is that you want to blow the boundary portions of the manifold out to infinity and work with the resulting noncompact manifold with cylindrical ends. Very notation-heavy and kind of dense, but it's also free from his website: http://www-math.mit.edu/~rbm.
- *Linear and Nonlinear Aspects of Vortices* by Frank Pacard and Tristan Riviére. This book purports to be about the Ginzburg0Landau problem, but it's also really an excuse to preach the virtues of weighted function spaces. It concentrates more on Hölder spaces than Sobolev spaces, but it has a very nice treatment of the fundamental problems and ideas. Unfortunately, you can't get it for free (that I know of).
- *The Null Spaces of Elliptic Partial Differential Operators in* $\mathbb{R}^n$ by Nirenberg and Walker, in J. Math. Anal. Appl., 1973. I think this was the first paper to seriously treat PDEs on weighted function spaces.
- *The Behavior of the Laplacian on Weighted Sobolev Spaces* by Robert McOwen, in Comm. Pure Appl. Math., 1979. Works with a polynomial weighting. From more of a potential theory/harmonic analysis perspective.
- *Elliptic Theory of Differential Edge Operators* by Rafe Mazzeo, Comm. PDE, 1991. Very general results; very dense.
- *Fredholm Operators and Einstein Manifolds on Conformally Compact Manifolds* by Jack Lee, arXiv preprint DG/0105046. The ideas lurking in conformally compact Einstein manifolds are very interesting. The idea is that you have a compact manifold with boundary and a metric that blows up at some specific rate near the boundary, pushing it off to infinity. Then you try to make the metric on the interior Einstein. The basic example is hyperbolic space (where the the conformal infinity is the conformal class of the round sphere). Anyhow, it's a pretty active area of research right now and it some involves a lot of analysis on these noncompact manifolds.