

Practice Problems
Math 252
February 8, 2006

1. Rewrite each of the following complex numbers in the form $z = a + ib$, where a and b are real numbers
 - (a) $z = \sqrt{1+i}$
 - (b) $z = e^{3+i\pi/2}$
 - (c) $z = [\cos(\pi/3) + i \sin(\pi/3)]^{1/3}$
2. Rewrite each of the following complex numbers in polar form $z = re^{i\theta}$, where $r \geq 0$ and $-\pi < \theta \leq \pi$.
 - (a) $1 - i$
 - (b) $[-\sqrt{3} + i]^{1/3}$
 - (c) $\sqrt{1 + e^{-i\pi/2}}$
3. Describe all the z which solve each of the following equations.
 - (a) $\log(1 + z) = i$
 - (b) $e^{z^2} = -2$
 - (c) $\sqrt{2z + 1} = 1 + i$
4. For each of the following functions $f(z)$ and domains D , describe the image domain $f(D)$.
 - (a) $f = z^3$, $D = \{z \mid 1 \leq |z| \leq 2, \pi/2 \leq \arg(z) \leq \pi\}$
 - (b) $f = \sin z$, $D = \{z = x + iy \mid 0 \leq x \leq \pi, y > 0\}$
5. Determine whether each of the functions below is analytic at the given point. Be sure to explain your answer.
 - (a) $f(z) = \frac{1+z}{1-z}$, $z_0 = 2$
 - (b) $f(z) = |z|^2$, $z_0 = i$
 - (c) $f(z) = u(x, y) + iv(x, y)$ where $u = x^2 - y^2$, $v = -2xy$, $z_0 = 0$
 - (d) $f(z) = \log(1 + z)$, $z_0 = 1$
6. Let $f(x + iy) = u(x, y) + iv(x, y)$ be an analytic function.
 - (a) Write out the complex derivative of f at $z_0 = 0$ in terms of partial derivatives of u and v , approaching along the ray $x = y > 0$.
 - (b) Do the same thing approaching along the ray $x = -y > 0$.
 - (c) Write out the resulting differential equations u and v must satisfy.
7. Consider the function $f(z) = z^2 - z$.
 - (a) Describe the infinitesimal behavior of f near $z_0 = 1 - i$.
 - (b) This function f is conformal near all points except one point z_1 . Find this point.
 - (c) Describe the infinitesimal behavior of f near this point z_1 .
8. Evaluate the following contour integrals.
 - (a) $\int_{\gamma} (1 + 3z^2) dz$, $\gamma(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi$
 - (b) $\int_{|z|=1} \frac{dz}{z}$
 - (c) $\int_{|z-3|=1} z^2 e^{3z} dz$