Math 2280: Computer Project 2
Due March 12

In this project you will analyze some of the factors that went into the collapse of the Tacoma Narrows bridge in 1940. There is a nice explanation of the physics behind the collapse at

http://www.math.utah.edu/ korevaar/2250fall03/tnarrows

The central differential equations here are the equations for the horizontal displacement $x(t)$ and the vertical displacement $y(t)$. These equations are

$$x'' + cx' + k\sin(x)\cos(x) = f(t) \tag{1}$$

and

$$y'' + cy' + \frac{k}{3}y = g. \tag{2}$$

Here c is the coefficient of the damping force (i.e. friction), k is the spring constant of the bridge, g is the acceleration of gravity and $f(t)$ is the driving force (in this case, wind). The URL listed above has a nice derivation of these formulas. (Remark: where is the mass? First of all, you should think of these equations as modeling a given section of the bridge, so the mass should really be a density. Then we can choose units so that we can assume the density is 1.)

For the actual project, you will model some free and forced oscillations using these equations and answer some questions about the models.

1. When x is small, equation (1) is approximately

   $$x'' + cx' + kx = f(t). \tag{3}$$

   Let's first fix $k = 25$ and $f = 0$. What is the value of c associated to critical damping? Have Maple (or whatever computer tool you're using) plot the solution to this equation ($k = 0, f = 0$) in each of the under-damped, over-damped and critically damped cases.

2. Now let's add a forcing term to equation (3) of the form

   $$f(t) = 10\cos(\omega t).$$

   If $c = 0$, what are the right initial conditions to have beats? Plot this solution. (You're going to have to choose a value for ω; it doesn't matter which value you choose.) Now make c very small, say $c = .0001$, with this same initial conditions. What happened? Plot the solution you get.

3. Answer the same question with ω closer to the natural frequency, say $\omega = 4.9$. (Be sure to include a plot of the solution!)

4. Ok, now let's look at the nonlinear model of equations (1, 2). This time we'll take $c = .05, k = 2.4, f(t) = .06\cos(6t/5)$, with the initial conditions $x(0) = 0, x'(0) = v_0$. Plot the solution to this initial value problem fro several values of v_0, say $v_0 = .05, v_0 = .5, v_0 = 1$. (Remember that $g = 9.8$.) You might want to plot $x(t)$ and $y(t)$ separately, for the sake of clarity.