Practice Problems

Math 2280

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1. For each of the following differential equations, decide whether the given function is a solution.
   (a) $y' = (x+1)(y^2-1)$, $y = -\frac{1+\exp(x^2+2x)}{1-\exp(x^2+2x)}$ (here $\exp(z) = e^z$)
   (b) $y' + 2xy = x$, $y = 2 - e^{x^2}$
   (c) $y' = xy$, $y = e^{x^2/2}$
2. Sketch the slope field and some typical solution curves for each of the following differential equations.
   (a) $y' = y^2 - 1$
   (b) $y' + xy = 0$
   (c) $y' = x^2 + y^2$
3. Solve each of the given initial value problems.
   (a) $y' - 2xy = x$, $y(0) = 1$
   (b) $y' - x^2 e^{-y} = 0$, $y(0) = 4$
   (c) $y' - ye^{yx} = 0$, $y(0) = 0$
   (d) $y' = y^2$, $y(0) = 1$
   (e) $y' + (1/x)y = x$, $y(1) = 1$
4. Consider the differential equation
   $$y' = y(y^2 - 1).$$
   (a) Find all the equilibria (constant solutions) of this equation.
   (b) Classify each of these equilibria as unstable, stable, or strictly stable.
   (c) Sketch some typical solution curves.
5. (a) Write down a differential equation with one strictly stable equilibrium. (Be sure to justify your answer.)
   (b) Write down a differential equation with one stable, but not strictly stable, equilibrium. (Be sure to justify your answer.)
6. Consider the differenital equation
   $$y' = y(y^2 - 1) - h,$$
   where h is a parameter.
   (a) Does this system have a bifurcation point? If it does, find the bifucation point.
   (b) Describe how the behavior of the system depends on h.
   (c) Sketch some typical solution curves, depending on the value of the parameter h.
7. Write down the differential equation which models the following phemonenom: the tangent line to the graph of any solution passes through the origin. Also, solve this differential equation with the initial conditions
   $$y(1) = 1.$$
8. Consider a falling body, where the force of wind resistance is proportial to the velocity, say
   $$F_R = -m\rho v,$$
   where m is the mass of the body, v its velocity (in the vertical direction) and $\rho > 0$ is a constant.
   (a) Write down a differential equation for v. You may assume that the only two forces acting on the body are gravity (which is a constant force of $-mg$) and wind resistance.
   (b) Find the constant solutions (i.e. equilibria) and classify them as unstable, stable, or strictly stable.
   (c) Sketch the slope field and some typical solution curves.
   (d) Explain your picture physically. In other words, what happens to this falling body?

9. For each of the following differential equations, say whether the superposition principle holds. (Be sure to justify your answer.)
   (a) $y'' + xy = x$
   (b) $y'' + x^2y' - xy = 0$
   (c) $y'' - y^2 = 0$
   (d) $y'' + \sin(x)y = \cos(s)y'$
10. Consider a differential equation of the form
$$y''' + p(x)y'' + q(x)y' + r(x)y = 0.$$
   (a) How many linearly independent solutions does this equation have? Why?
   (b) Given three solutions of this equation, how can you tell that they are linearly independent? Why does this work?
11. Consider a differential equation of the form
$$y'' + p(x)y' + q(x)y = 0,$$
and let y_1, y_2 be two solutions of this equation. Recall that the Wronskian is
$$W := y_1y_2' - y_2y_1'.$$
   (a) The Wronskian W satisfies a differential equation. What is it?
   (b) Conclude from this differential equation that $W = W(x)$ is either never zero or always zero.
12. Solve the following initial value problems.
   (a) $y'' - 3y' + 4y = 0$, $y(0) = 1, y'(0) = 0$
   (b) $y'' - y = 0$, $y(0) = -1, y'(0) = 1$
   (c) $y'' + 5y' + 6y = 0$, $y(0) = 1, y'(0) = 1$