

Computer Project II
Math 2270-1, April 2002

In the first part of this project, you will compute have MAPLE (or whichever computing environment you prefer) compute some Fourier coefficients for various functions. This could be quite useful, for instance, if you ever want to do some signal processing. Below I have the an inner product suited to Fourier series and some of the appropriate functions typed in. You should fill in the blanks where they occur.

```
> restart:with(plots):  
dotprod := (f,g) -> int(f(t) * g(t), t = -Pi..Pi);  
Warning, the name changecoords has been redefined
```

$$\text{dotprod} := (f, g) \rightarrow \int_{-\pi}^{\pi} f(t) g(t) dt$$

```
> f_1 := t ->(1/sqrt (Pi)) * cos (t):  
f_2 := t ->(1/sqrt (Pi)) * cos (2*t):  
f_3 := t ->(1/sqrt (Pi)) * cos (3*t):  
f_4 := t ->(1/sqrt (Pi)) * cos (4*t):  
f_5 := t ->(1/sqrt (Pi)) * cos (5*t):  
f_0 := t ->1/sqrt(2*Pi):  
> g_1 := t ->(1/sqrt(Pi)) * sin (t):  
g_2 := t ->(1/sqrt(Pi)) * sin (2*t):  
g_3 := t ->(1/sqrt(Pi)) * sin (3*t):  
g_4 := t ->(1/sqrt(Pi)) * sin (4*t):  
g_5 := t ->(1/sqrt(Pi)) * sin (5*t):
```

These functions above are the first 11 elements of the orthonormal basis we found for this inner product we are using.

- (1) Have MAPLE (or whichever program you are using) verify that f_1 and g_1 are perpendicular.
- (2) Have MAPLE (or whatever) verify that f_1 and f_2 are perpendicular.
- (3) Have MAPLE (or whatever) verify that f_1 and g_1 are unit length.
- (4) Have MAPLE (or whatever) plot f_1, f_2, g_1 and g_2 on the same set of axes so you can look at them.

```
> h := t -> t^2;
```

$$h := t \rightarrow t^2$$

```
>
```

Now we will compute a truncated Fourier series for this function h. Below you will do the same thing for other functions.

```
> a_0 := dotprod (f_0,h);  
a_1 := dotprod (f_1,h);  
a_2 := dotprod (f_2, h);  
a_3 := dotprod (f_3,h);  
a_4 := dotprod (f_4,h);  
a_5 := dotprod (f_5, h);
```

$$a_0 := \frac{1}{3} \sqrt{2} \pi^{(5/2)}$$

$$a_1 := -4 \sqrt{\pi}$$

$$a_2 := \sqrt{\pi}$$

$$a_3 := -\frac{4}{9} \sqrt{\pi}$$

$$a_4 := \frac{1}{4} \sqrt{\pi}$$

$$a_5 := -\frac{4}{25} \sqrt{\pi}$$

These are the first six even Fourier coefficients for h.

```
> b_1 := dotprod (g_1,h);
  b_2 := dotprod (g_2,h);
  b_3 := dotprod(g_3,h);
  b_4 := dotprod (g_4,h);
  b_5 := dotprod (g_5,h);
```

$$b_1 := 0$$

$$b_2 := 0$$

$$b_3 := 0$$

$$b_4 := 0$$

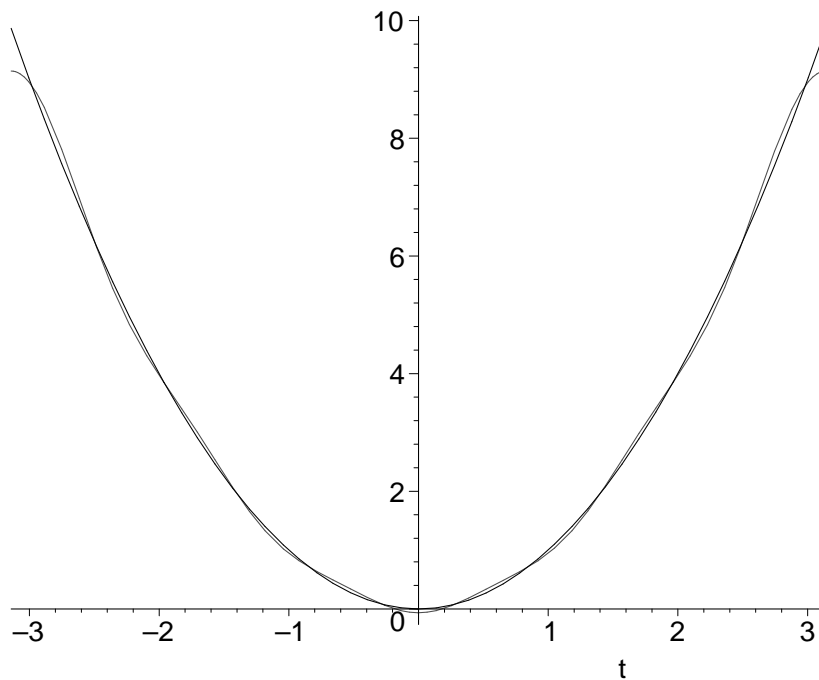
$$b_5 := 0$$

These are the first five odd Fourier coefficients for h. You could have predicted that they would all be zero (why?).

```
> H := t -> a_0* f_0(t) + a_1*f_1(t) + a_2*f_2(t) + a_3*f_3(t) +
  a_4*f_4(t) + a_5*f_5(t) + b_1*g_1(t) + b_2*g_2(t) + b_3*g_3(t) +
  b_4*g_4(t) + b_5*g_5(t) ;
```

```
H := t -> a_0 f_0(t) + a_1 f_1(t) + a_2 f_2(t) + a_3 f_3(t) + a_4 f_4(t) + a_5 f_5(t) + b_1 g_1(t)
  + b_2 g_2(t) + b_3 g_3(t) + b_4 g_4(t) + b_5 g_5(t)
```

```
> plot1 := plot (h(t), t = -Pi..Pi, color = black):
  plot2 := plot (H(t), t = -Pi..Pi, color = red):
  display ({plot1,plot2});
```



This plot shows the original function h (in black) and its truncated Fourier series H (in red) on the same plot. Notice that they are very close. Also notice that they agree very well near zero, but not so well near the endpoints. This phenomenon is called Gibbs' phenomenon.

```
>
> h_1 := t -> t:
  h_2 := t -> abs(t):
  h_3 := t -> (abs(t))/t:
```

Here are some functions. You are probably familiar with h_1 ; h_2 looks something like a sawtooth wave (near $t = 0$) and h_3 looks something like a block wave (again, near $t = 0$). The second two represent functions which occur frequently in wave analysis and signal processing.

(5) Have MAPLE (or whatever) plot these three functions so you can see what they look like.

(6) Have MAPLE (or whatever) compute the first 11 Fourier coefficients for h_1 by computing the inner product of h_1 with f_0, \dots, f_5 and with g_1, \dots, g_5 .

(7) Have MAPLE (or whatever) compute the truncated Fourier series of h_1 (i.e. have it compute the orthogonal projection of h_1 onto the span of f_0, \dots, f_5 and g_1, \dots, g_5).

(8) Have MAPLE (or whatever) plot h_1 and the truncated Fourier series you just computed on the same axes so you can compare them. You might want to also compare h_1 with a further truncated Fourier series (say, the projection onto f_0, f_1, f_2, g_1, g_2) in the same manner.

(9) Do the same thing for h_2 and h_3 .