

Practice Problems
Math 2270
January 23, 2002

1. Classify the following functions from \mathbb{R} to \mathbb{R} as linear, affine or other.

- (a) $f(x) = 2x + 1$
- (b) $g(x) = x^2$
- (c) $h(x) = 3x$

2. Consider the system of linear equations

$$\begin{aligned} x + y + z &= 2 \\ x - z &= 1 \\ x - y &= -1. \end{aligned}$$

- (a) Each of the linear equations yields a plane in \mathbb{R}^3 . Find the normal for each plane and a point on each plane.
- (b) Write down the augmented matrix for this system.
- (c) Row reduce the matrix you wrote down in the previous part and find all solutions to this system. How many are there?
- (d) Verify that the solution you found yields a vector perpendicular to the normal vectors to all three planes.

3. Consider a general linear map $A : \mathbb{R}^n \rightarrow \mathbb{R}^m$.

- (a) This map is given by matrix multiplication. What is the size of the matrix.
- (b) If A is one to one, what can you say about the relationship between n and m ?
- (c) Now let $n = m = 2$ and change coordinates in \mathbb{R}^2 by rotating counterclockwise through an angle of $\pi/4$. How can you relate the matrix of A in the new coordinates to the matrix of A in the old (usual) coordinates?

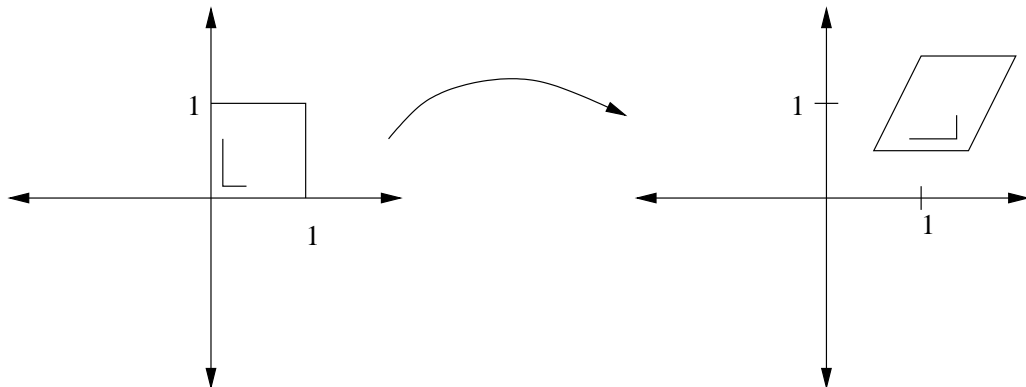
4. We will consider affine maps $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ in this problem.

(a) Let $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the affine map

$$F(x) = \begin{bmatrix} 0 & -1/3 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1/3 \\ 0 \end{bmatrix}.$$

Draw a picture of the image of the unit square $\{0 \leq x \leq 1, 0 \leq y \leq 1\}$ under F . Be sure to indicate which corner is the image of $(0,0)$ and the orientation of the image.

(b) Consider the parallelogram drawn below, which is the image of the unit square as indicated. Recover the affine map which generates this picture.



5. Explain the relationship between a matrix A being invertible and the ability to solve the equation $Ax = b$ for x . In particular, remark on when you can solve this equation for any right hand side b , and on when you get a unique solution x to this equation.

6. Let $A : \mathbb{R}^2 \rightarrow \mathbb{R}^2$. Give a necessary and sufficient geometric condition for A to be invertible. (Hint: think about the image of the unit square under A .)

7. Let B be the matrix

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & 1 & 0. \end{bmatrix}$$

- (a) Find the inverse of B by row reducing.
 - (b) Multiply B by its inverse (in either order) and verify that you get the identity matrix.
 - (c) Solve the equation $Bx = v$ where $v = (1, 0, 2)$ using the inverse you found for B .
8. Let A and B be 2×2 matrices. Is it true that $(A+B)^2 = A^2 + 2AB + B^2$? If it is true, explain why. Otherwise, find a counterexample and find a formula for $(A+B)^2$ in terms of A and B .