Letters to the Editor

The Fibonacci Sequence: Relationship to the Human Hand

To the Editor:

It was my great good fortune to spend a year in the early 1970s with Dr. J. William Littler and Dr. Richard Eaton as one of their hand fellows. During that time Dr. Littler was thinking about the geometry of the hand, which led to his well-known article, "On the Adaptability of Man's Hand (with reference to the equiangular curve)."[1] Because of our shared interest in mathematics Dr. Littler and I had numerous enjoyable discussions about the Golden Mean, Fibonacci's sequence, and equiangular spirals throughout that year. Consequently I was pleased to see the article by Park et al, "The Fibonacci Sequence: Relationship to the Human Hand" (J Hand Surg 2003; 28A:157–160). However, it appears to me that Fibonacci's sequence, the ratio of the Golden Mean (ϕ), equiangular spirals, and the Chambered Nautilus have been linked together so often in various articles that their interrelationships have become misunderstood. This letter is an attempt to clarify the ways in which they are and are not interconnected.

The equiangular (logarithmic, geometric, proportional, spira mirabilis) spiral is not one single specific spiral curve but is an infinite set of spirals, all with the same fundamental properties but differing in characteristic variables, and approaching a circle at one limit and a straight line at the other (Fig. 1).

If from a center point (polar axis) a radius line of constant, unchanging length is rotated, a circle results. If the radius increases steadily in length at a rate exactly proportional to its rate of rotation an equiangular spiral results, enlarging endlessly until the radius rotation stops. Any given equiangular spiral has several interrelated parameters, any one of which identifies that particular spiral: the proportion of length of increasing radius to angle of sweep of the radius is constant; the angle between a tangent to the curve at any point and the radius line at that point is a constant, hence the name *equiangular*; the size increases at a constant rate of growth but the shape is unchanged, which is the feature that relates to spiral shells such as the snail and the nautilus. The equiangular spiral uniquely related to the chambered nautilus is one such spiral (Fig. 2), and the equiangular spiral of the Golden Mean, mathematically related to the Fibonacci series, is a different spiral (Fig. 3).

The Golden Mean is a number with many special properties. It is the proportion defined by the relationship between 2 unequal straight lines such that the ratio of the shorter to the longer is exactly the same as the ratio of the longer to the sum of the 2 lengths, $a/b = b/(a + b)$. This results in only one number, called ϕ, an endlessly long decimal number whose value is 1.61803. . . . . to 5 places. A rectangle whose sides are in this ratio seems to most people to be of particularly pleasant proportions and is termed a *Golden Rectangle*. Two different isosceles triangles have sides in this ratio, a shorter one with angles 108/36/36, and a taller one, sometimes referred to as the Golden Triangle, with angles 36/72/72.

If a series of nested Golden Rectangles, or of Golden Triangles, is constructed, and equivalent points on the periphery of each successive component (gnomon) are joined, an equiangular spiral will be generated (Fig. 4), as seen in Dr. Littler's illustrations and in the article by Park et al. The ratio of

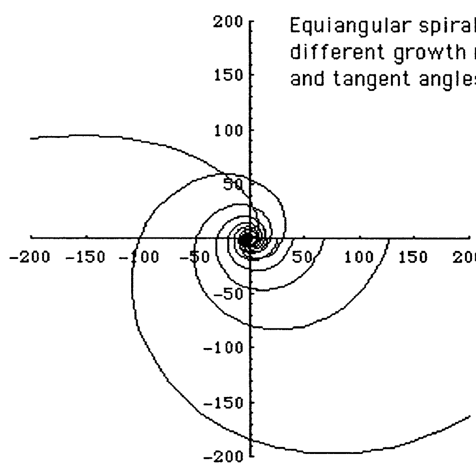

Figure 1. Equiangular spirals with different growth rates and tangent angles.

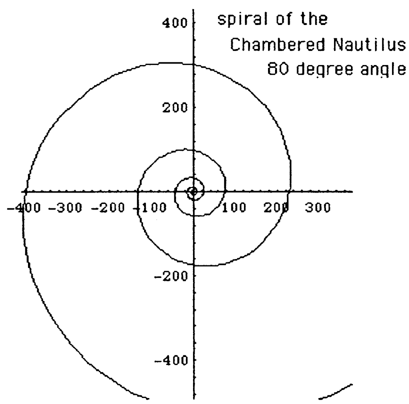

Figure 2. Spiral of the Chambered Nautilus 80° angle.

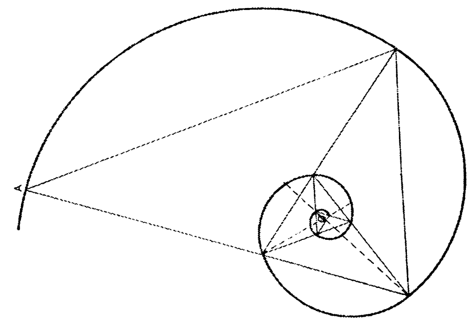

Figure 4. Equiangular spiral generated by nested gnomonic triangles.

the generating radius will be ϕ, 1.618.... The unique tangent angle of this spiral is 72.9°. An equiangular spiral also is generated by any other set of nested non–Golden Mean rectangles or triangles of equivalent shape but increasing/decreasing size (gnomons), but the radius proportion and tangent angle will be different from the Golden Mean spiral. The shell of the Chambered Nautilus has a tangent angle of 80° and a proportion ratio of 1.318, a smaller number and tighter curve than the spiral of ϕ.

The Fibonacci series is a sequence of integers, starting with 0 and 1, and proceeding such that the next number is equal to the sum of the preceding 2, thus 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so forth. The ratio of each 2 adjacent numbers, 1/1, 1/2, 2/3, 3/5, 5/8, 8/13, and so forth, approach but never quite reach the value of ϕ closer and closer in an oscillating manner, to infinity (Fig. 5). It is not the case, as stated in the study by Park et al that "the ratio of any 2 consecutive numbers (of the Fibonacci sequence) equals ϕ." The ratio approaches ϕ as a limit. The Fibonacci sequence is the simplest of a group of series called the Lucas series, such that one can start with any 2 integers and proceed so that the next number is equal to the sum of the preceding 2, and the resulting series also will always approach ϕ as the limit.

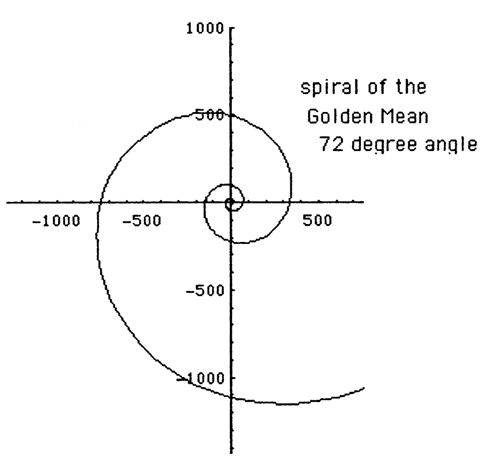

Figure 3. Spiral of the Golden Mean 72° angle.

The study by Gupta et al referred to in the article by Park et al confirmed Dr. Littler's observation that the normal unrestrained arc of flexion and extension of the digits of the human hand closely follow equiangular spiral curves. The study by Gupta et al does not report the tangent angles or proportionality ratios observed in their study. Given the variation in all other measurement parameters of human anatomy and function it is likely that the digital arcs lie in a range clustered around the 73° tangent angle Golden Mean spiral. Indeed, direct measurement of this angle in a variety of Dr. Littler's figures show a range from 71° to 75°.

The article by Park et al primarily deals with the hypothesis that the equiangular nature of the digital flexion arc is explained by phalangeal lengths related as in Fibonacci's series. Their data show poor correlation for this hypothesis. They speculate, I believe correctly, that a more likely correlation should be with the functional, center of rotation lengths. In-

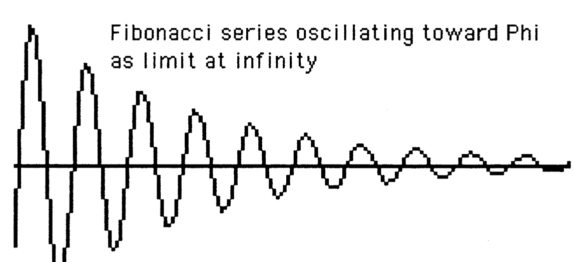

Figure 5. Fibonacci series oscillating toward ϕ as limit at infinity.

deed, Dr. Littler in his article states "Carpo-metacarpophalangeal and interphalangeal interaxial links determine this curve; their relationships approach closely the ratio 1/1.618." The ratios of mean digital interaxial lengths noted in his figures range from 1/1.54 to 1/1.64. Thus it is likely that the radius vectors determining the equiangular sweep of digital flexion are a reflection of functional interaxial lengths approaching Fibonacci or Lucas series ratios.

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doi:10.1016/S0363-5023(03)00251-X

Reference

1. Littler JW. On the adaptability of man's hand (with reference to the equiangular curve). Hand 1973;5:187–191.

In Reply:

We appreciate the detailed descriptions of the Golden Mean Φ, Fibonacci sequence, and equiangular spiral that Dr. Markley so elegantly presents. This would have been a very useful appendix to the article. We hope the readers find this helpful in understanding the mathematic complexities of these relationships and their application to the human hand.

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Journal Club Official Selection

To the Editor:

The Southern California Society for Surgery of the Hand recently held its annual journal club meeting during which we discussed clinical articles from the *Journal of Hand Surgery,* volume 27A. As initiated last year, we select what we think is the best article and recognize the authors with a letter and honorarium. We select an article that our membership feels is thought provoking, possibly controversial, and has the potential to alter our thinking with regard to diagnosis or treatment of a particular entity. This process takes our membership through a comprehensive dissection of excellent articles. We offer our selection to the *Journal of Hand Surgery*'s readership as one worthy of their attention, both for its present and potential value. From volume 27A we have selected "Collagen as a Clinical Target: Nonoperative Treatment of Dupuytren's Disease" by Drs. Badalamente, Hurst, and Hentz as the best clinical contribution for the year. Our congratulations to the authors for publishing this fine article and to the *Journal of Hand Surgery* for providing professional enhancement.

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doi:10.1016/S0363-5023(03)00205-3

Erratum

In the article by Catalano et al entitled "Treatment of Chronic, Traumatic Hyperextension Deformities of the Proximal Interphalangeal Joint With Flexor Digitorum Superficialis Tenodesis," which appeared in the May 2003 issue (Vol 28A, No 3, pp 448–452), the spellings of two co-authors' names had been submitted incorrectly to the *Journal.* The correct spellings of their names are Debra Mulley, MD, and Lewis B. Lane, MD.