The Fibonacci Sequence: Relationship to the Human Hand

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Purpose: The motion path of the digits follows the path of an equiangular spiral in which a constant angle is formed by all radial vectors along the curve. This implies that the lengths of the metacarpals, proximal, middle, and distal phalanges approximate a Fibonacci sequence in which the ratio of any 2 consecutive numbers approaches the number 1.61803 (phi). This study tested the hypothesis that the metacarpal and phalangeal bone lengths follow the Fibonacci relationship.

Methods: Standardized x-rays were taken of the hands of 100 healthy volunteers. The proximal phalanx length was subtracted from the sum of the lengths of the middle and distal phalanges and the metacarpal length was subtracted from the sum of the lengths of the middle and proximal phalanges. Confidence intervals for the quotients of the measured lengths of the adjacent bones of the hand also were used for statistical analysis.

Results: Only 1 of 12 bone length ratios contained the ratio phi in the 95% confidence interval, that of the small finger metacarpal and proximal phalanx. The largest variability was seen in the small finger phalangeal relationships.

Conclusion: The application of the Fibonacci sequence to the anatomy of the human hand, although previously accepted, is a relationship that is not supported mathematically. The difference between individual bone lengths as measured at the joint line and the center of rotation of the joints may explain our finding. (J Hand Surg 2003;28A:157–160. Copyright © 2003 by the American Society for Surgery of the Hand.)

Key words: Fibonacci, anatomy, equiangular spiral, bone lengths.

Harriot described the equiangular spiral, the Spira Mirabilis, in 1590.[1] In an equiangular spiral a constant angle is formed by all radial vectors along the curve (Fig. 1). This relationship was based originally on the mathematical series first described by Leonardo Pisano Bigollo (also known as Fibonacci). This series is defined by a sequence of numbers generated by the sum of the previous 2 numbers (0,1,1,2,3,5,8,13, 21, and so forth). Thompson's[2] biologic work, *On Growth and Form*, made observations of naturally occurring logarithmic curves in the shell of the nautilus, in the egg, and in the movement of the digits. Littler[3] reiterated that the motion path of digits follows an equiangular spiral. He also inferred that the bone lengths would follow the Fibonacci relationship,[3,4] whose sequence dictates that the ratio of any 2 consecutive numbers, for example, metacarpal/proximal phalangeal length, equals 1.62 (phi). Gupta et al[5] experimentally confirmed that the motion path of the fingers does follow the equiangular spiral by using a motion analysis system. Our study evaluates the mathematic validity of the Fibonacci relationship with respect to the bone lengths of the human hand. Although this anatomic relationship generally has been accepted to be true,[3,6] it has not been specifically studied.

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Materials and Methods

One hundred consecutive posteroanterior views of skeletally mature hands were collected from 100

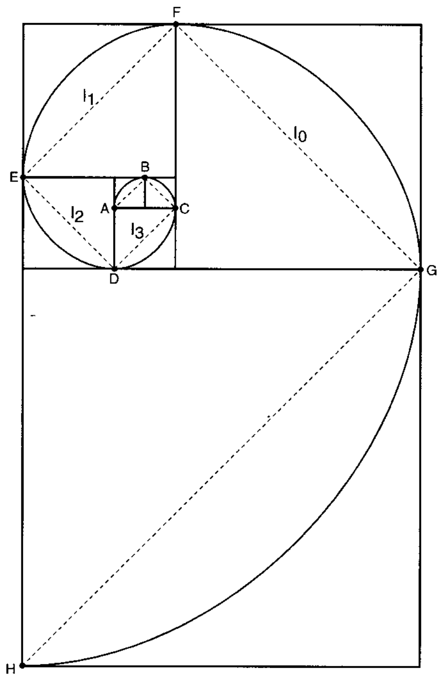

Figure 1. Diagram depicting the relationship between the Fibonacci sequence, the Fibonacci rectangles, and the equiangular spiral. The sides of each successive square follow the Fibonacci relationship, as do the diagonals of the squares defined by connecting points A through H. For example, I3 + I2 = I1; I2 + I1 = I0, and so forth.

healthy volunteer subjects. Exclusion criteria included prior fracture or surgery of the hand, any congenital skeletal anomaly, skeletal immaturity, or joint contracture. All radiographs were taken by a single technologist using a standardized technique (Fig. 2). The radiographic beam was centered on the middle finger metacarpophalangeal joint. Two independent examiners evaluated radiographs. Measurements were performed to the nearest half-millimeter by using Vernier calipers (Cole-Parmer, Chicago, IL). Each data field was composed of a letter to indicate the digit and a number to indicate the bone. Investigational Review Board approval was obtained. The data were maintained and analyzed on a Microsoft Excel (Seattle, WA) program.

The length of the proximal phalanx was subtracted from the sum of the lengths of the distal and middle phalanges and the length of the metacarpal was subtracted from the sum of the middle and proximal phalanges for each digit. Because the next element in Fibonacci's sequence is determined by the sum of the prior 2 numbers, the difference should equal zero if the bone lengths of the human hand correlated mathematically with the Fibonacci sequence. The 95% confidence intervals were calculated for the index through small fingers.

Quotients of lengths of adjacent finger bones also were calculated for comparison with the Fibonacci ratio. The 95% confidence interval limits are given by the standard expression:

$$\bar{x} \pm t_{.025,n-1} \frac{s}{\sqrt{n}}$$

where $\bar{x}$ equals the average quotient of 2 particular bone lengths among all hands, s is the corresponding SD, n is the sample size, and $t_{.025,n-1}$ equals the inverse value of the relevant student's *t* distribution (with $n-1$ degrees of freedom and tail probabilities of .025).

The ratio of the lengths of the metacarpals and

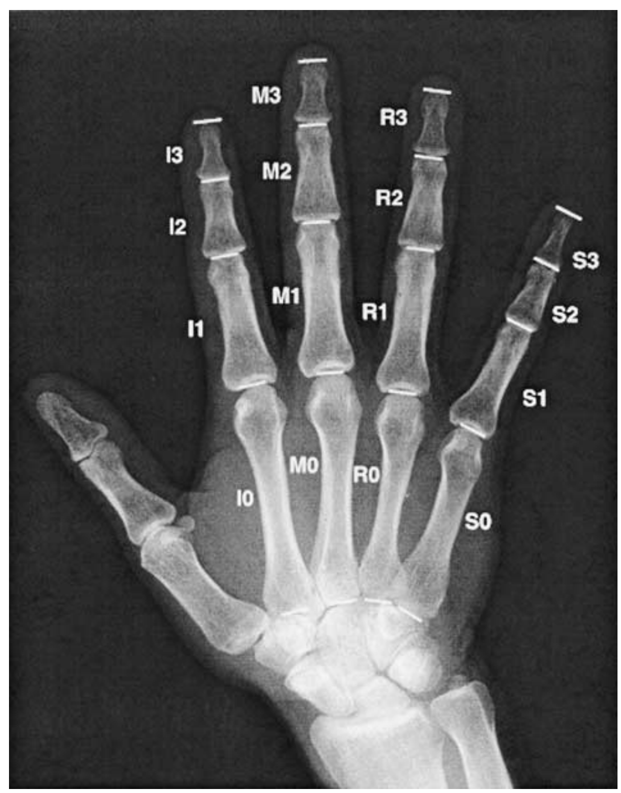

Figure 2. Posteroanterior radiograph of a hand. Bone lengths identified by the name of the field used for statistical analysis. Each data field is composed of a letter to indicate the digit and a number to indicate the bone: I, index finger; M, middle finger; R, ring finger; S, small finger; 0, metacarpal; 1, proximal phalanx; 2, middle phalanx; 3, distal phalanx.

Table 1. Confidence Intervals

	95% Confidence Interval	$(P_3 + P_2) - P_1$	$(P_2 + P_1) - MC$
Index	Lower limit	0.42	−5.34
	Upper limit	1.15	−3.46
Middle	Lower limit	1.64	6.90
	Upper limit	2.33	8.30
Ring	Lower limit	3.56	9.74
	Upper limit	4.25	11.08
Small	Lower limit	2.64	−2.13
	Upper limit	4.06	−0.62

P_3, proximal phalanx; P_2, middle phalanx; P_1, proximal phalanx; MC, metacarpal.

phalanges were calculated separately for each digit. The 95% confidence intervals with the upper and lower limits were calculated for each quotient.

Results

The 95% confidence intervals for the difference of the proximal phalanx from the sum of the middle and distal phalanges and the difference of the metacarpal from the sum of the middle and proximal phalanges are provided in Table 1. Zero is not contained in the 95% confidence interval for any of the relationships in any of the digits.

The quotients of the adjacent bone lengths are presented in Table 2. The ratio (phi) of the Fibonacci sequence is contained in the 95% confidence interval only for the quotient of the small finger metacarpal and proximal phalanx. The other 11 confidence intervals for the remaining digits did not contain phi.

The difference between the upper and lower limits of the calculated quotients was greatest in the small finger. This difference ranged from .03 to .04 for all quotients involving the index, middle, and ring fingers. For the small finger quotients this difference ranged from .05 to .12 ($p < .05$) (Table 2).

Discussion

The lengths of the metacarpals and phalanges are related intimately to their motion path in flexion and extension. For their motion to follow the equilateral spiral, the phalangeal and metacarpal bone lengths would theoretically need to follow the Fibonacci relationship because the 2 are mathematically linked by the Fibonacci rectangles (Fig. 1). Leonardo Pisano first described the Fibonacci sequence in the 13th century.[7] In this sequence each number is defined by the sum of the 2 preceding numbers (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, and so forth). This relationship was first used to describe the population growth of rabbits assuming a constant reproductive rate. The Fibonacci relationship has proved to be quite fruitful in many applications in nature as well as mathematics. Other naturally occurring phenomena for which the sequence applies include the chambered nautilus, pine cones, and branching and flowering plants (please see appendix A on the Journal's Web site, www.jhandsurg.com). As the ratio of 2 successive numbers in Fibonacci's series is taken to its infinite term, this ratio, called phi (ϕ), has a value of 1.61803.

To extrapolate this relationship to the bony anatomy of the hand, the sum of the distal phalanx and middle phalanx should be equal to the length of the proximal phalanx in a given digit. Similarly, the sum of the middle phalanx and proximal phalanx should be equal to the length of the metacarpal. The findings of the present study refute such a relationship because the bone lengths do not follow the Fibonacci sequence. When subtracting the proximal phalangeal bone length from the sum of the distal and middle phalanges, zero is not contained within the 95% confidence interval for any of the digits. This also is true for the difference of the metacarpal length to that of the sum of the middle and proximal phalanges (Table 1).

With the exception of the quotient of the metacarpal to the proximal phalanx of the small finger, the ratio Phi is never contained in the 95% confidence interval. The largest intervals are those involving the small finger phalanges (Table 2). The SDs of the quotients for the small finger are substantially greater than those

Table 2. Confidence Intervals for Ratios

	95% Confidence Interval	P_2/P_3	P_1/P_2	MC/P_1
Index	Lower limit	1.33	1.70	1.67
	Upper limit	1.37	1.73	1.72
Middle	Lower limit	1.52	1.57	1.45
	Upper limit	1.56	1.60	1.48
Ring	Lower limit	1.42	1.54	1.38
	Upper limit	1.45	1.57	1.42
Small	Lower limit	1.11	1.68	**1.61***
	Upper limit	1.18	1.80	**1.66***

P_3, proximal phalanx; P_2, middle phalanx; P_1, proximal phalanx; MC, metacarpal; Phi (ϕ) ≈ 1.61803.
*These quotients were the only ones contained in the 95% confidence interval.

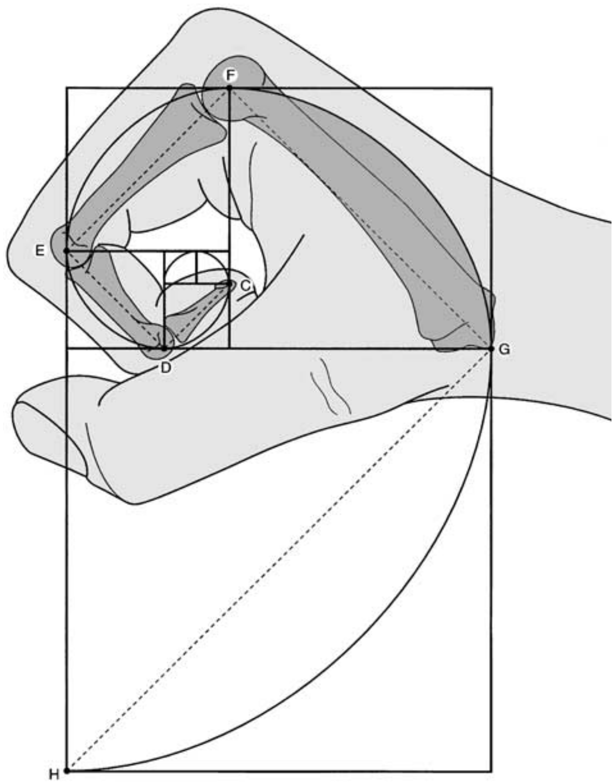

Figure 3. Human hand superimposed on the Fibonacci rectangles and equiangular spiral of Figure 1. This shows the proposed relationship of the Fibonacci sequence to the center of rotation of the joints of the hand. The connection of points C to D, D to E, and so forth represents the diagonals of the Fibonacci squares. Points D, E, and F depict the center of rotation of the metacarpophalangeal, proximal, and distal interphalangeal joints, respectively.

of the other digits, suggesting more variation in the development of the postaxial skeleton; the functional implications of this cannot be determined by the present study. It is known, however, that the anatomy of the metacarpophalangeal joint of the small finger differs from that of the other digits.[8] This anatomic difference has been shown to result in a moving center of rotation for the small finger metacarpophalangeal joint in flexion and extension.[7] In addition, the small finger carpometacarpal joint has a greater degree of motion and variability than that of the adjacent digits.

At first glance it is difficult to understand why the bone lengths of the fingers do not follow the Fibonacci relationship, whereas the motion paths of the digits form an equiangular spiral. On closer inspection of the bony anatomy, however, it becomes evident that absolute bone lengths may be less important that the centers of rotation of the digit. We hypothesize that the distance from the base of the metacarpal to the center of rotation of the metacarpophalangeal joint to the center of rotation of the proximal and distal interphalangeal joints should yield lengths that do follow the Fibonacci relationship (Fig. 3). It is not logical to assume that the center of rotation of each digit falls in the clear space between the joints with the fingers fully extended. This difference between absolute bone length (measured at the joint line) and functional length (determined by the center of rotation of the joints) may explain this study's paradox.

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