Third Project: Math 223
Due April 22, 2005

This project will explore tilings of the Euclidean plane. A *tiling* is a repeated pattern of shapes that covers the plane without overlapping or leaving gaps. For instance, the unit squares with integer vertices tile the plane.

1. Recall that a regular n-gon is a polygon such that all the all its sides are congruent, and all its interior angles are congruent.
   (a) Find the measure (in either degrees or radians) of the interior angle of a regular n-gon. (Hint: connect the center to the vertices.)
   (b) We call a tiling regular if it's made from copies of a single regular n-gon, with all the n-gons meeting at a common vertex. If a regular tiling by n-gons has k polygons meeting at each vertex, what can you say about n and k? (Hint: add up the angles.)
   (c) Which regular n-gons provide regular tilings of the plane?
2. Now let's relax the condition and allow two regular n-gons.
   (a) Find a tiling of the plane using a square and an equilateral triangle. Describe it as best you can; you might want to draw a picture.
   (b) Find another tiling of the plane using equilateral triangles and regular hexagons. Describe it as best you can; you might want to draw a picture.
   (c) Which other tilings can you find using two regular polygons? Describe them as best you can; you might want to draw pictures. Can you find conditions on the number of vertices in your pair of regular polygons?
3. Now let's only use one polygon but relax the condition on the polygon. In particular, we will not require that the polygon is regular.
   (a) Show that all triangles provide regular tilings of the plane.
   (b) Not all quadrilaterals provide regular tilings of the plane. Find one that doesn't, and argue why it doesn't.
   (c) Which quadrilaterals provide regular tilings of the plane. (Hint: in a way, you've already found these tilings.)
4. It turns out there are regular tilings by irregular polygons! Roger Penrose discovered these in the late 1970's. These tilings are not periodic. (*I.e.* they are not invariant under any translation; the tilings we've found so far are periodic.) Describe the Penrose's kites and darts, and some possible ways they tile the plane.