Practice Problems

Math 223

Feb. 9, 2005

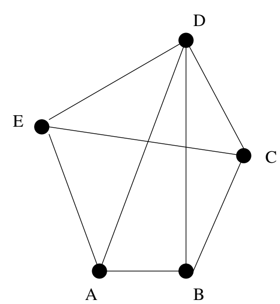
These problems are in no particular order.

1. For each of Euclid's propositions listed below, draw a picture to illiustrate them and translate them to modern language.

   (a) If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.

   (b) If with any straight line, and at a point on it, two straight lines not lying on the same side make the sum of the adjacent angles equal to two right angles, then the two straight lines are in a straight line with one another.

2. Consider the following model with points and lines given. So the points are A, B, C, D, E and the lines are



   $\{A, B\}, \{B, C\}, \{C, D\}, \{D, E\}, \{E, A\}, \{A, D\}, \{B, D\}, \{E, C\}$.

   (a) Does this satisfy the incidence axioms? List the axioms it satisfies and the one it doesn't. Be sure to explain your answer.

   (b) Regardless of whether it does satisfy the incidence axioms, we can still find parallel lines. What are all the lines parallel to $\{E, C\}$ and incident to D?

3. (a) Prove that all three-point incidence geometries are isomorphic. (Hint: write out the mapping of points; this forces a certain mapping of lines.)

   (b) Given a three-point incidence geometry, how many isomorphisms are there from it to itself? (Hint: how many ways can you re-order three things?)

4. Prove ASA congruence of triangles given SAS congruence of triangles.

5. Prove that Playfair's axiom is equivalent to the converse to the alternate interior angles theorem.

6. Justify the steps in showing that one can construct an angle measuring $135°$, using a straight-edge and a compass.

7. Consider the triangle $\triangle ABC$, and let AD bisect the angle $\angle CAB$, with $D \in BC$. Prove that AD meets BC at a right angle if and only is $AB \simeq AC$. (Hint: congruent triangles. You may use the fact that $AB \simeq AC$ if and only if $\angle ABC \simeq \angle ACB$, and that the angle sum of a triangle is $180°$. You really don't need to use the angle sum part, but it might make the proof easier.)