# Practice Problems

Math 223

Sept. 17, 2004

These problems are not in any particular order. The exam will be shorter. As in the last exam, I will list the axioms you should know.

1. Recall Euclid's fifth postulate: if a transversal $t$ intersecting two lines $l$ and $m$ so that the angle sum of the interior angles on one side of $t$ is less than $180°$, then the lines $l$ and $m$ intersect, and the intersection point lies on the same side of $t$ as both these angles.

   (a) State the converse to Euclid's fifth postulate.

   (b) Prove the converse to Euclid's fifth postulate.

   (c) **Assuming Euclid's fifth postulate**, prove that if the angle sum of the two interior angles on one side of $t$ is greater than $180°$, then the lines $l$ and $m$ cannot intersect on that side of $t$.

2. Recall that a Saccheri quadrilateral $\square ABCD$ satisfies $Ad \simeq BC$ and $(\angle DAB)^\circ = 90^\circ = (\angle CBA)^\circ$.

   (a) Prove that $\angle ADC \simeq \angle BCD$.

   (b) Let $M$ be the midpoint of $AB$ and $M'$ the midpoint of $CD$. Prove that $MM'$ is perpendicular to both $AB$ and $CD$.

   (c) Prove $|MM'| < |AD|$. (Hint: in a triangle, larger sides are opposite larger angles.)

3. Consider a triangle $\triangle ACD$ where $\angle ACD$ is a right triangle, and let $B$ be a point so that $A * B * C$. Prove that $AD > BD > CD$. You might want to draw a picture.

4. Recall that the defect of a triangle $\triangle ABC$ is $\delta ABC = 180^\circ - (\angle A)^\circ - (\angle B)^\circ - (\angle C)^\circ$. In the Euclidean plane, all triangles have the same defect (namely, $0^\circ$). Do all triangles in the hyperbolic plane have the same defect? Explain your answer. (Hint: when you sub-divide a triangle, the defects add.)

5. Let $l, l'$ be distinct parallel lines, with $M \in l$ and $M' \in l$. Prove that if $MM'$ is perpendicular to both $l$ and $l'$ then it is the shortest segment joining $l$ and $l'$.

6. Prove that the diagonals of a convex quadrilateral intersect. (Hint: the crossbar theorem.)

7. The convex hull of a set of points is the smallest convex set containing all of them. Prove that the convex hull of three points $A, B, C$ is the triangle with vertices $A, B, C$. What are the possibilities for the convex hull of four points?

8. Recall Hilbert's parallel postulate: given line $L$ and $p \notin L$, there is at most one line $M$ containing $p$, parallel to $L$. Suppose Hilbert's parallel postulate is true, and let $k, l, m, n$ be lines. Prove that if $k \| l$, $m \perp k$ and $n \perp l$ then either $m = n$ or $m \| n$.