Practice Problems

Math 223

Sept. 17, 2004

These problems are not in any particular order. The exam will be shorter. I will list all of Hilbert's axioms (which include those of incidence geometry) on the exam.

1. Negate the following statements.
   (a) P and Q
   (b) P implies Q
   (c) Every line contains at least three points.
   (d) Given a triangle with vertices A, B, C such that $AB \simeq AC$, the two angles with vertices B and C are congruent.
   (e) If two distinct line l and m intersect, they have only one point in common.
2. Prove or find a counter-example: All four-point incidence geometries are isomorphic.
3. Does the following model satisfy the axiom of incidence geometry?

4. Consider the following model for an incidence geometry.

   (a) Does this model satisfy the Euclidean parallel postulate?
   (b) How many automorphisms (i.e. self-maps which are isomorphisms) does this model have?
5. Consider the following model for an incidence geometry.

   (a) Show this is a projective plane.

(b) Draw a model for the dual projective plane. Be sure to write a sentence or two explaining your answer.

6. Prove that any projective plane has at least seven points.

7. Prove that two triangles with congruent sides are congruent, using Hilbert's axioms. (Hint: congruence axiom six tells you SAS holds, so you're done if you can show SSS implies SAS.)

8. Let $\vec{AD}$ be a ray between the rays $\vec{AB}$ and $\vec{AC}$. Prove that $\vec{AD}$ intersects the segment BC.

9. Prove that if $A * B * C$ then $\vec{BA} cap \vec{BC} = \{B\}$.

10. Let $\mathbb{Q}^2$ be the ordered pairs of rational points:

$$\mathbb{Q}^2 := \{(p, q) : p, q \text{ ar rational}\}.$$

(This is called the rational plane. Show that the first congruence axiom and the elementary continuity axioms both fail.