Practice Problems

Math 211

March 22, 2006

1. Consider the system

$$\begin{cases} \frac{dx_1}{dt} &= x_1 + x_2 \\ \frac{dx_2}{dt} &= x_2. \end{cases}$$

   (a) Is $x_1(t) = te^t, x_2(t) = e^t$ a solution? Explain your answer.

   (b) Is $x_1(t) = e^{2t}, x_2(t) = e^{-t}$ a solution? Explain your answer.

2. Find the equilibrium (i.e. constant) solutions of the system

$$\begin{cases} \frac{dx_1}{dt} &= x_1 - 2x_1x_2 \\ \frac{dx_2}{dt} &= x_1x_2 - x_2. \end{cases}$$

3. Consider the second order differential equation

$$\frac{d^2x}{dt^2} + t\frac{dx}{dt} + t^2x = 0.$$

   (a) Rewrite this as a 2×2 system of first order equations.

   (b) Verify that the only equilibrium solution is $x_1 = 0, x_2 = 0$.

4. Consider the system

$$\begin{cases} \frac{dx_1}{dt} &= x_1 + x_2 \\ \frac{dx_2}{dt} &= 2x_1 - x_2 \end{cases}$$

   (a) Rewrite this system in matrix form:

$$\frac{d\vec{x}}{dt} = A\vec{x},$$

   where A is a 2×2 mattrix.

   (b) Find the eigenvalues and eigenvectors of A.

   (c) Write down the general solution of the system.

   (d) Solve the initial value problem for this system with the initial condition $x_1(0) = 2, x_2(0) = -1$.

   (e) Is the origin a stable or unstable equilibrium?

   (f) Sketch some typical solution curves to the system in the phase plane.

5. Consider the system

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \vec{x}.$$

   (a) Find the eigenvalues and eigenvectors for the coefficient matrix A for this system.

   (b) Is the origin a stable or unstable equilibrium?

   (c) Solve the inital value problem with the initial condition $x_1(0) = 1, x_2(0) = -1$.

   (d) Write down the general solution to this system.

   (e) Sketch some of the solution curves in to this system in the phase plane.

6. Consider the system

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 1 & 2 \\ -1 & a \end{bmatrix} \vec{x},$$

   where a is a real parameter.

   (a) Find the eigenvalues in terms of a.

   (b) Sketch the phase portrait of the system near the origin when $a = 0$ and when $a = -1$.

   (c) For which values of a are there straight line solutions? (Hint: what do the solutions look like when the eigenvalues are complex numbers?)

   (d) For which values of a is the origin a stable equilibrium?

7. Consider the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0.$$

(a) Rewrite this equation as a first order system.

(b) Find the general solution to this equation.

(c) Solve the initial value problem with $x(0) = -1, x'(0) = 1$.