

Practice Problems

Math 211

February 8, 2006

1. For each of the following differential equations, decide whether the given function is a solution.

(a) $\frac{dx}{dt} = (t+1)(x^2 - 1)$, $x = -\frac{1+\exp(t^2+2t)}{1-\exp(t^2t)}$ (here $\exp(z) = e^z$)

(b) $\frac{dx}{dt} + 2xt = t$, $x = 20e^{t^2}$

(c) $\frac{dx}{dt} = xt$, $x = e^{t^2/2}$

2. Sketch the slope field and some typical solution curves for each of the following differential equations.

(a) $\frac{dx}{dt} = x^2 - 1$

(b) $\frac{dx}{dt} + x^2 - t^2 = 0$

(c) $\frac{dx}{dt} = x^2 + t^2$

3. Solve each of the given initial value problems.

(a) $\frac{dx}{dt} - 2xt = t$, $x(0) = 1$

(b) $\frac{dx}{dt} - t^2e^{-x} = 0$, $x(0) = 4$

(c) $\frac{dx}{dt} - xe^x = 0$, $x(0) = 0$

(d) $\frac{dx}{dt} = x^2$, $x(0) = 1$

(e) $\frac{dx}{dt} + x/t = t$, $x(1) = 1$

4. Consider the differential equation

$$\frac{dx}{dt} = x(x^2 - 1).$$

(a) Find all the equilibria (constant solutions) of this equation.

(b) Classify each of these equilibria as sinks, sources or nodes

(c) Sketch some typical solution curves.

5. Consider the differential equation

$$\frac{dx}{dt} = ax - x^2.$$

(a) Draw the slope field and sketch some representative solution curves for the parameter values $a = 1, 0, -1$.

(b) Find the equilibrium solutions for the parameter values $a = 1, 0, -1$.

(c) Classify these equilibria as sinks, sources, or nodes.

(d) Does this differential equation have a bifurcation point? Find the bifurcation point if it exists. Be sure to explain your answer.

6. (a) Write down a differential equation with two equilibria, one of which is a sink and one of which is a source. (Be sure to justify your answer.)

(b) Write down a differential equation with equilibrium, which is a node. (Be sure to justify your answer.)

7. Consider the differential equation

$$\frac{dx}{dt} = x(x^2 - 1) - a,$$

where a is a parameter.

(a) Does this system have a bifurcation point? If it does, find the bifurcation point.

(b) Describe how the behavior of the system depends on a .

(c) Sketch some typical solution curves, depending on the value of the parameter a .