

Practice Problems
Math 210
Oct. 19, 2006

These problems are not in any particular order. The exam will be shorter (about or 5 problems).

1. Consider the function $f(x, y) = x^3y - yx^2 + 3xy$.

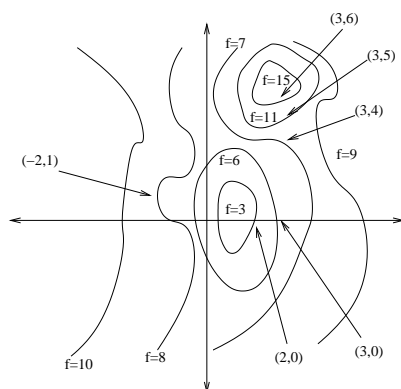
- (a) Write down the equation of the tangent plane to the graph of f at $(0, 1)$.
- (b) At which points (x, y) is the tangent plane horizontal?

2. Consider

$$f(x, y) = \int_x^y e^{t^2} dt.$$

- (a) Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
- (b) Does f have any critical points? Explain your answer.

3. Let some of the level sets of the function f be given by the figure below.



- (a) Estimate $\frac{\partial f}{\partial x}(2, 0)$
- (b) Estimate $\frac{\partial f}{\partial y}(3, 4)$
- (c) In which direction does $\nabla f(-2, 1)$ point?
- (d) Suppose you know $(1, 0)$ is a critical point. Would you guess it's a local maximum, a local minimum, or neither? Explain your answer.

4. Consider $f(x, y) = xe^{x^2+y^2}$ and let $(x_0, y_0) = (1, 1)$.

- (a) Compute $\nabla f(1, 1)$.
- (b) Find the directional derivative $\nabla f \cdot \vec{u}(1, 1)$, where $\vec{u} = (1/2, \sqrt{3}/2)$.
- (c) What is the direction of steepest increase for f , starting at $(1, 1)$. (Be sure to write down a unit vector!)
- (d) Notice $f(1, 1) = e$. What is the equation of the tangent line to the level set $\{f = e\}$, at the point $(1, 1)$?

5. Consider $f(x, y) = \cos x \cos y$.

- (a) Classify all the critical points of f .
- (b) Find the absolute maximum and minimum of f on the square $\pi/4 \leq x \leq 3\pi/4, \pi/4 \leq y \leq 3\pi/4$.

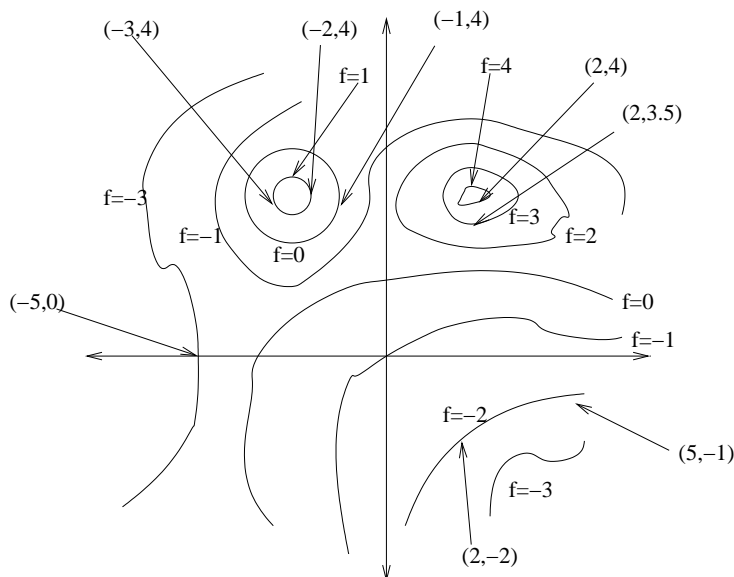
6. Consider the composition $F(t) = f(x(t), y(t))$, where f is a function of the two variables x and y , while x and y are both functions of t .

- (a) Suppose $x(1) = 0, y(1) = 2, x'(1) = 2, y'(1) = -1, \frac{\partial f}{\partial x}(0, 2) = 5$, and $\frac{\partial f}{\partial y}(0, 2) = -3$. Compute $F'(1)$.
- (b) If $x'(0) = 0$ and $y'(0) = 0$, is it true that $F'(0) = 0$? Explain your answer.
- (c) If $F'(3) = 0$, is it true that $x'(3) = 0$ and $y'(3) = 0$? Explain your answer.

7. Compute the slope of the tangent line to the hyperbola $x^2 - y^2 = 1$ at the point $(2, \sqrt{3})$.
8. (a) Evaluate $\int_0^1 \int_0^1 [x^2y + yx^3] dx dy$.
 (b) Evaluate $\int_{x^2+y^2 \leq 1} [e^{x^2+y^2}] dx dy$. (Hint: try polar coordinates.)
 (c) Evaluate $\int_D [xy - x^2y^2] dx dy$, where D is the ice-cream cone shaped region bounded by $y = x - 1$, $y = -x - 1$, and $x^2 + y^2 = 1$. (It might help to draw a picture of D).
9. Find the minimum of $f = y^3 - 3yx^2$ on the ellipse $x^2 + 4y^2 \leq 1$.
10. Consider the function

$$f(x, y) = \int_{2-y}^x \sqrt{1+t^2} dt.$$

- (a) Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
 (b) Does f have any critical points? Be sure to explain your answer.
11. Consider the following sketch of level curves of the function $f(x, y)$.



- (a) Estimate $\frac{\partial f}{\partial y}(2, 3.5)$.
 (b) In which direction does $\nabla f(-5, 0)$ point? Be sure to explain your answer.
 (c) If f has a critical point at $(-2.5, 4.5)$, do you expect it to be a local minimum, a local maximum, or a saddle point? Be sure to explain your answer.
12. Consider the function

$$f(x, y) = x^4 + y^4 - 4xy + 1.$$

- (a) Verify that the critical points of f are $(0, 0)$, $(1, 1)$, and $(-1, -1)$.
 (b) Classify these critical points as local maxima, local minima, or saddle points.
13. Find the absolute maximum of $f(x, y) = xy$ on the ellipse $g(x, y) = x^2 + 4y^2 \leq 1$.
14. (a) Where D is the square $\{1 \leq x \leq 2, -2 \leq y \leq -1\}$, evaluate

$$\iint_D [x^2y + yx^3] dA.$$

- (b) Set up, but do **not** evaluate, the integral $\iint_D \sqrt{1+x^2+y^2} dA$, where D is the domain bounded by the curves $y = x + 1$ and $x = -y^2$.

15. Consider the function

$$f(x, y) = \int_{1-y}^{x^2} \frac{dt}{1+t^2}.$$

- (a) Compute the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$.
- (b) Does f have any critical points? Be sure to explain your answer.

16. Consider the function

$$f(x, y) = x^2 + y^3 - 6xy.$$

- (a) Verify that the only critical points of f are $(0, 0)$ and $(18, 6)$.
- (b) Classify each of these two critical points as local minima, local maxima, or saddle points.
- (c) Observe that $f(1, 1) = -4$. Find the equation of the tangent line to the level set $\{f = -4\}$ at the point $(1, 1)$.
- (d) Compute $D_{\vec{u}}f(1, 1)$, where $\vec{u} = (\sqrt{3}/2, 1/2)$.

17. Let $F = F(u, v)$, while $u = u(t)$ and $v = v(t)$. Call the resulting composition $f(t) = F(u(t), v(t))$.

- (a) If $u(0) = 1$, $u'(0) = 2$, $v(0) = -1$, $v'(0) = 3$ and $\nabla F(1, -1) = (8, -1)$, then what is $f'(0)$?
- (b) If $u'(1) = 0$ and $v'(1) = 0$ then is it always true that $f'(1) = 0$? Be sure to explain your answer.
- (c) If $f'(-1) = 0$, is it always true that $u'(-1) = 0 = v'(-1)$? Be sure to explain your answer.

18. Find the maximum and minimum of $f(x, y) = xy$ subject to the constraint $x^4 + y^4 = 1$.

19. (a) Evaluate $\int \int_D [x^2y]dA$ where $D = [1, 2] \times [2, 3]$.

- (b) Set up, but **do not** evaluate $\int \int_D e^{1-x^2-y^2} dA$, where D is bounded by $x = y^2 - y$ and $y = x/2$. (Hint: it might help to draw a picture.)

20. Consider the two circle given in polar coordinates coordinates by $\{r = 2\}$ and $\{r = 4 \sin \theta\}$.

- (a) Sketch these two circles and find their points of intersection.
- (b) Find the area lying inside the circle $\{r = 4 \sin \theta\}$ but outside the circle $\{r = 2\}$. (Hint: the area is given by a double integral. You might find some of the trigonometric identities on the front useful.)