Solutions to the Practice Problems

Math 210

Sept. 25, 2006

1. Given the following pairs of vectors $\vec{u}$ and $\vec{v}$, find the angle θ between them and compute the cross product $\vec{u} \times \vec{v}$.

   (a) $\vec{u} = (1, 2, 3)$, $\vec{v} = (-2, 1, 0)$
   The angle θ is given by
   $$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = 0,$$
   so the angle is $\theta = \pi/2$. Next we compute the cross product:
   $$\vec{u} \times \vec{v} = (0 - 3, -6 - 0, 1 + 4) = (-3, -6, 5).$$

   (b) $\vec{u} = (1, 0, 2)$, $\vec{v} = (2, 1, 0)$
   The angle θ is given by
   $$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{2}{5},$$
   so the angle is $\theta = \cos^{-1}(2/5) \simeq 1.16$. Next we compute the cross product:
   $$\vec{u} \times \vec{v} = (0 - 2, 4 - 0, 1 - 0) = (-2, 4, 1).$$

   (c) $\vec{u} = (1, 1, 0)$, $\vec{v} = (1, 0, 1)$
   The angle θ is given by
   $$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{1}{5},$$
   so the angle is $\theta = \cos^{-1}(1/5) \simeq 1.37$. Next we compute the cross product:
   $$\vec{u} \times \vec{v} = (1 - 0, 0 - 1, 0 - 1) = (1, -1, -1).$$

2. Consider the plane Π_1 containing $p = (2, 1, 3)$, with normal vector $\vec{n} = (-1, 2, 0)$.

   (a) Write down the linear equation any point (x, y, z) in this plane must satisfy.
   The linear equation of Π_1 is given by
   $$0 = \vec{n} \cdot ((x, y, z) - p) = (-1, 2, 0) \cdot (x - 2, y - 1, z - 3) = -x + 2y.$$

   (b) Find the angle between the plane Π_1 and the plane Π_2 determined by $x - y = 2$.
   The angle between Π_1 and Π_2 is the same as the angle between the two normal vectors $\vec{n}$ and $\vec{m} = (1, -1, 0)$. This angle θ is given by
   $$\cos\theta = \frac{\vec{n} \cdot \vec{m}}{\|\vec{n}\|\|\vec{m}\|} = -\frac{3}{\sqrt{10}},$$
   so $\theta = \cos^{-1}(-3/\sqrt{10}) \simeq= 2.82$

   (c) Parameterize the line l which is the intersection of Π_1 and Π_2.
   We can parameterize l as $l(t) = q + \vec{v}t$, where q is a point common to both planes Π_1 and Π_2 and $\vec{v}$ is lies in the same direction as l. We could compute $\vec{v}$ by computing the cross product $\vec{n} \times \vec{m}$, but in this case there's an easy trick: observe that both $\vec{n}$ and $\vec{m}$ lie in the $x - y$ plane, so they are both perpendicular to the vector $\vec{v} = (0, 0, 1)$.
   Now we have to find a point q in both Π_1 and Π_2. The coordinates (x, y, z) of q will satisfy the system
   $$x - y = 2 \qquad -x + 2y = 0.$$
   Simultaneously solving for x and y we see $x = 4, y = 2$. What is z? It can be anything. One can see that from the fact that l is parallel to the z-axis. We choose $z = 0$, so $q = (4, 2, 0)$ and
   $$l(t) = (4, 2, 0) + t(0, 0, 1) = (4, 2, t).$$

(d) Find the distance between the plane Π_1 and the point $q = (3, 3, 3)$.

We can compute the distance by computing the orthogonal projection of $q-p = (3, 3, 3)-(2, 1, 3) = (1, 2, 0)$ onto $\vec{n} = (-1, 2, 0)$. This orthogonal projection is

$$\vec{a} = (\cos\theta)|q-p|\frac{\vec{n}}{|\vec{n}|} = \frac{(q-p)\cdot\vec{n}}{|\vec{n}|}\frac{\vec{n}}{|\vec{n}|} = \frac{3}{5}(-1, 2, 0).$$

The distance is the length of this orthogonal projection

$$d = |\vec{a}| = \frac{3}{5}\sqrt{1+4} = \frac{3}{\sqrt{5}}.$$

3. Consider the vectors $\vec{u} = (1, 2, 1)$ and $\vec{v} = (0, 1, -1)$.

(a) Explain why all the planes parallel to both $\vec{u}$ and $\vec{v}$ will have the same normal vectors (up to scaling).

All parallel planes have the same normal vector (up to scaling), because any plane is given by $\vec{n}\cdot((x, y, z)-(x_0, y_0, z_0))$, for some choice of of normal vector $\vec{n}$ and point (x_0, y_0, z_0) on the plane. Changing which plane you pick in a family of parallel planes amounts to varying (x_0, y_0, z_0) (in the direction of $\vec{n}$). Thus they all have the same normal vector. Another way to think of this is: the angle between two planes is the angle between the two normal vectors. So two planes are parallel precisely when their normal vectors are parallel, which is another way of saying the normals are the same up the scaling.

(b) Are these planes parallel to the plane given by $x + y + z = 2$? Explain your answer.

The normal vector to the plane spanned by $\vec{u}$ and $\vec{v}$ is

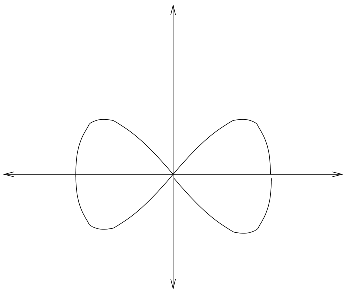
$$\vec{n} = \vec{u}\times\vec{v} = (-3, 1, 1),$$

while the plane $x + y + z = 3$ has the normal vector $\vec{m} = (1, 1, 1)$. These normal vectors are not parallel, so the two planes are not parallel.

4. Consider the plane curve given by $c(t) = (\cos(t), \sin(2t))$, for $0 \le t \le 2\pi$.

(a) Sketch this curve.

The curve c is a sideways figure 8, with the twist in the 8 at the origin. Here is a sketch.



(b) Set up, but do not evaluate, the integral to compute the arclength of c.

length $= \int_0^{2\pi} |c'(t)|dt = \int_0^{2\pi} \sqrt{\sin^2(t) + 4\cos^2(2t)}dt.$

(c) Notice c is periodic $(c(0) = c(2\pi))$. Is c a simple closed curve? In othre words, are the t parameters 0 and 2π the only times c crosses itself?

No, c crosses itself at the origin, which corresponds to $t = \pi/2$ and $t = 3\pi/2$.

5. (a) Consider the right circular cone C, with vertex at $(0, 0, 0)$, and slope 1. In other words, the cone C is what you get when you rotate the line $y = z$ in the $y-z$ plane about the z-axis. Write C in cylindrical coordinates.

For a given height z, the distance from the z-axis is also z and the angle can be anything. So in cylindrical coordinates, this is $r = z$, with θ unrestricted.

(b) Write the part of the shell $1 \le x^2 + y^2 + z^2 \le 4$ lying in the $x < 0, y > 0, z < 0$ octant in spherical coordinates.

First, $x^2 + y^2 + z^2 = \rho^2$, so we have $1 \le \rho \le 2$. Next we have to find bounds on the angles. The quadrant $x < 0$, $y > 0$ in the $x-y$ plane corresponds to $\pi/2 \le \theta \le \pi$. Also, $z < 0$ means we restrict $\pi/2 \le \phi \le \pi$. So this region is

$$1 \le \rho \le 2, \quad \pi/2 \le \theta \le \pi, \quad \pi/2 \le \phi \le \pi.$$

6. Consider the space curve $c(t) = (\cos(t), \sin(t), t)$.

   (a) Is the velocity vector ever tangent to the x-axis?
   The velocity is

$$c' = \frac{d}{dt}(\cos t, \sin t, t) = (-\sin t, \cos t, 1).$$

   In order for c' to be tangent to the x-axis, its last two components have to vanish. However, the last component is 1, so this never happens.

   (b) Verify the Fundamental Theorem of Calculus by checking

$$c(2\pi) - c(0) = \int_0^{2\pi} c'(t)dt.$$

$$c(2\pi) - c(0) = (\cos(2\pi), \sin(2\pi), 2\pi) - (\cos(0), \sin(0), 0) = (1, 0, 2\pi) - (1, 0, 0) = (0, 0, 2\pi).$$

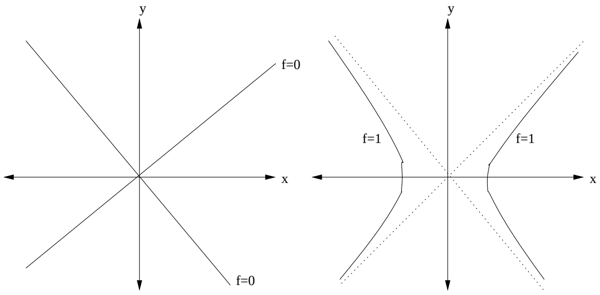
   Meanwhile,

$$\int_0^{2\pi} c'(t)dt = \int_0^{2\pi} (-\sin t, \cos t, 1)dt = \left(\cos t|_0^{2\pi}, \sin t|_0^{2\pi}, t|_0^{2\pi} \right) = (0, 0, 2\pi).$$

   (c) Set up, but do not evaluate, the integral to compute the arclength of c for $0 \leq t \leq 2\pi$.
   length $= \int_0^{2\pi} |c'(t)|dt = \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t + 1}dt = \int_0^{2\pi} \sqrt{2}dt = 2\pi\sqrt{2}$.

7. Consider the function $f(x, y) = x^2 - y^2$.

   (a) Sketch the level sets $f = 0$ and $f = 1$.
   The level set $f = 0$ is given by $x^2 = y^2$, or $x = \pm y$. This is the union of two lines, crossing at right angles at the origin. The level set $f = 1$ is the hyperbola $x^2 - y^2 = 1$. Here is a sketch.



   (b) Does f have an upper bound? How about a lower bound?
   f has neither an upper bound nor a lower bound. Indeed, by setting $y = 0$, we see that $f(x, 0) = x^2$, which can be as large and positive as you please. Similarly, $f(0, y) = -y^2$, which can be as large and negative as you please.

   (c) Compute the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$.
   We have

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2) = 2x$$

   and

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 - y^2) = -2y.$$

   (d) Is the tangent plane to the graph of f ever parallel to the $x - y$ plane?
   The tangent plane has normal vector

$$\vec{n} = (1, 0, \partial f/\partial x) \times (0, 1, \partial f/\partial y) = (1, 0, 2x) \times (0, 1, -2y) = (-2x, 2y, 1).$$

   Also, the $x - y$ plane has normal $(0, 0, 1)$, which is parallel to $\vec{n}$ (in fact, equal to $\vec{n}$) precisely when $x = y = 0$.

8. Explain why the tangent plane to the graph of a function $f(x, y)$ cannot ever be parallel to the $x - z$ or $y - z$ planes, provided f has continuous partial derivatives.

   The tangent plane to the graph of f is spanned by $(1, 0, \partial f/\partial x)$ and $(0, 1, \partial f/\partial y)$, so it has the normal vector

$$\vec{n} = (1, 0, \partial f/\partial x) \times (0, 1, \partial f/\partial y) = (-\partial f/\partial x, -\partial f/\partial y, 1).$$

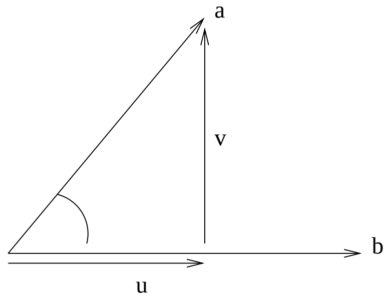
   However, the normal vector to the $y-z$ plane is $(1, 0, 0)$, which cannot ever be parallel to $(-\partial f/\partial x, -\partial f/\partial y, 1)$. Similar reasoning applies to the $x - z$ plane.

9. Consider the vectors $\vec{a} = (2, 1, 3)$ and $\vec{b} = (-1, 0, 1)$.

   (a) (3 points) Compute the cross product $\vec{a} \times \vec{b}$.
   $\vec{a} \times \vec{b} = (2, 1, 3) \times (-1, 0, 1) = (1 - 0, -3 - 2, 0 + 1) = (1, -5, 1)$.

   (b) (3 points) Find a vector $\vec{x}$ which is perpendicular to $\vec{a}$ and verify that $\vec{x} \perp \vec{a}$. (There are many correct answers.)
   You want to find $\vec{x}$ so that $\vec{x} \cdot \vec{a} = 0$. Notice that $\vec{x} = (-1, 2, 0)$ works.

   (c) (4 points) Write $\vec{a}$ as a sum $\vec{a} = \vec{u} + \vec{v}$ where $\vec{u}$ is parallel to $\vec{b}$ and $\vec{v}$ is perpendicular to $\vec{b}$. (Hint: you only need to find one of $\vec{u}$ and $\vec{v}$. It might help to draw a picture.)
   Here's the picture you might have drawn:



   Ok, so this picture indicates

$$\begin{aligned}\vec{u} &= |\vec{a}|\cos(\theta)\frac{\vec{b}}{|\vec{b}|} = |\vec{a}|\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\frac{\vec{b}}{|\vec{b}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b} \\ &= \frac{(2,1,3)\cdot(-1,0,1)}{2}(-1,0,1) = (-1/2, 0, 1/2).\end{aligned}$$

   Then

$$\vec{v} = \vec{a} - \vec{u} = (5/2, 1, 5/2).$$

10. Consider the curve $c(t)$ given by

$$c(t) = (t\cos t, t\sin t, t).$$

   (a) (3 points) Find the velocity and acceleration vectors of this curve.
   The velocity is

$$c'(t) = \frac{d}{dt}(t\cos t, t\sin t, t) = (\cos t - t\sin t, \sin t + t\cos t, 1),$$

   and the acceleration is

$$c''(t) = \frac{d}{dt}(c') = \frac{d}{dt}(\cos t - t\sin t, \sin t + t\cos t, 1) = (-2\sin t - t\cos t, 2\cos t - t\sin t, 0).$$

   (b) (4 points) Is the tangent line to c ever parallel to the $x - y$ plane? Be sure to explain your answer.
   In order for the tangent line to c to be parallel to the $x-y$ plane, we would need c' to be a horizontal vector. In other words, we need the third component of c' to be 0. This never happens; the third component of c' is 1. So the tangent line to c is never parallel to the $x - y$ plane.

   (c) (3 points) Set up, but do not evaluate, the integral to compute the arclength of c for $0 \le t \le \pi$.

$$\begin{aligned}\text{length} &= \int_0^\pi |c'(t)|dt = \int_0^\pi \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2 + 1}dt \\ &= \int_0^\pi \sqrt{\cos^2 t - 2t\cos t\sin t + t^2\sin^2 t + \sin^2 t + 2t\cos t\sin t + t^2\cos^2 t + 1}dt = \int_0^\pi \sqrt{2 + t^2}dt.\end{aligned}$$

11. Consider the planes Π_1 and Π_2, given as follows. The first plane Π_1 passes through $p = (1, 2, 3)$ and has the normal vector $\vec{n} = (1, 0, -1)$. The second plane Π_2 is given by the linear equation $x + y + z = 1$.

    (a) (5 points) Explain how one can tell that Π_1 and Π_2 are not parallel, and compute the cosine of the angle θ between them.

    The normal vector to Π_1 is $\vec{n} = (1, 0, -1)$ and the normal vector to Π_2 is $\vec{m} = (1, 1, 1)$. These two vectors are not parallel, so the planes are not parallel If θ is the angle between them, then

$$\cos\theta = \frac{\vec{n} \cdot \vec{m}}{|\vec{n}||\vec{m}|} = \frac{(1, 0, -1) \cdot (1, 1, 1)}{\sqrt{2}\sqrt{3}} = 0.$$

    (b) (5 points) The two planes Π_1 and Π_2 intersect in a line l. Find a parameterization for l.

    First observe that the equation of Π_1 is

$$x - z = (1, 0, -1) \cdot (1, 2, 3) = -2.$$

    To find the line, we need a point and a direction. Let's find the point first, by looking for simultaneous solutions of

$$x - z = -2 \qquad x + y + z = 1.$$

    First we try setting $y = 0$, which gives us

$$x - z = -2 \qquad x + z = 1.$$

    Now we add the two equations, which gives us $2x = -1$, or $x = -1/2$, and so $z = 3/2$. Thus a point on the line l is $q = (-1/2, 0, 3/2)$. Now we can find a direction by taking the cross product of the two normal vectors:

$$\vec{v} = \vec{n} \times \vec{m} = (1, 0, -1) \times (1, 1, 1) = (1, -2, 1).$$

    Finally, we can parameterize the line by

$$l(t) = q + t\vec{v} = (-1/2, 0, 3/2) + t(1, -2, 1).$$

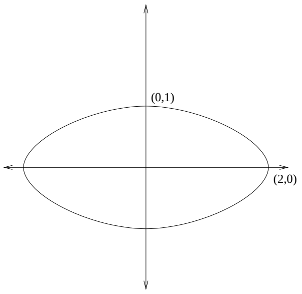
12. Consider the function $f(x, y) = x^2 + 4y^2$.

    (a) (5 points) Sketch the $f = 4$ level set.

    We can rewrite the $f = 4$ level set as

$$1 = \frac{1}{4}(x^2 + 4y^2) = \frac{x^2}{4} + y^2,$$

    which one can recognize as an ellipse with horizontal axis 2 and vertical axis 1. Here is a sketch.



    (b) (5 points) For which values of z does the level set $f = z$ not contain any points? Be sure to explain your answer.

    We want to find values of z such that the equation

$$x^2 + 4y^2 = z$$

    has no solutions. The left hand side can be zero, or any positive number, but it can't be negative. So the level sets $f = z$ for any negative z do not contain any points, while all other level sets contain at least one point.

13. Consider the function $f(x, y) = x^2y + y^2x$.

(a) (5 points) Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2y + y^2x) = 2xy + y^2 \quad \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2y + y^2x) = x^2 + 2xy$$

(b) (5 points) Find the points (x, y) where the tangent plane to the graph of f is parallel to the $x - y$ plane. Be sure to explain your answer.

We know that the two tangent directions of the graph of f are

$$(1, 0, \frac{\partial f}{\partial x}) = (1, 0, 2xy + y^2)$$

and

$$(0, 1, \frac{\partial f}{\partial y}) = (0, 1, x^2 + 2xy)$$

so the normal is given by their cross product, which is

$$\vec{n} = (1, 0, 2xy + y^2) \times (0, 1, x^2 + 2xy) = (-2xy - y^2, -x^2 - 2xy, 1).$$

So the tangent plane to the graph is f is horizontal precisely when the partial derivatives both vanish:

$$2xy + y^2 = 0 = x^2 + 2xy.$$

We can rearrange this to read

$$-y^2 = 2xy = -x^2 \Leftrightarrow x = 0 = y.$$

So the only solution is $x = 0, y = 0$.

14. Consider the two vectors

$$\vec{v} = (3, -2, 1) \qquad \vec{u} = (2, 1, 4).$$

(a) (4 points) Compute $\vec{u} \times \vec{v}$.

$$\vec{u} \times \vec{v} = (2, 1, 4) \times (3, -2, 1) = (1 - (-8), 12 - 2, -4 - 3) = (9, 10, -7)$$

(b) (4 points) Find a vector $\vec{w}$ which is perpendicular to $\vec{v}$, and explain why the answer you give is correct.

There are many possible choices. You could choose $\vec{w} = \vec{u} \times \vec{v} = (9, 10, -7)$, which you just computed. Or you could choose something to make the dot product zero, like $\vec{w} = (2, 3, 0)$ or $\vec{w} = (0, 1, 2)$. All these answers are equally correct.

15. Consider the planes Π_1 and Π_2 , where Π_1 contains the point $p_1 = (3, 2, 1)$ and has the normal vector $\vec{n_1} = (2, 0, 1)$, while Π_2 is given by the equation $x + y - z = 3$.

(a) (4 points) Find the cosine of the angle θ between Π_1 and Π_2.

The angle θ between the planes is the same as the angle between the two normals $\vec{n_1}$ and $\vec{n_2}$. The cosine is given by the dot product:

$$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = \frac{(2, 0, 1) \cdot (1, 1, -1)}{\sqrt{4 + 0 + 1}\sqrt{1 + 1 + 1}} = \frac{1}{\sqrt{15}}.$$

(b) (3 points) Write down the equation for Π_1.

The equation of the plane is given by

$$\vec{n_1} \cdot (x, y, z) = \vec{n_1} \cdot (3, 2, 1) \Rightarrow 2x + z = 7.$$

(c) (4 points) Find a vector $\vec{v}$ which is parallel to both Π_1 and Π_2.

We want $\vec{v}$ to be parallel to both Π_1 and Π_2, so we must have $\vec{v}$ perpendicular to both $\vec{n_1}$ and $\vec{n_2}$. One such vector is the cross product:

$$\vec{v} = \vec{n_1} \times \vec{n_2} = (2, 0, 1) \times (1, 1, -1) = (-1, 3, 2).$$

(d) (4 points) Parameterize the line l of intersection between Π_1 and Π_2.

To parameterize a line, we need to find two things: a direction vector and a basepoint. We just found the direction vector $\vec{v} = (-1, 3, 2)$ is the previous part (it's parallel to both planes), so all that remains is to find a basepoint, which is a simultaneous solution to the two equations

$$2x + z = 7 \qquad x + y - z = 3.$$

Notice y doesn't appear in the first equation, so let's try setting $y = 0$. Then we get

$$2x + z = 7, \qquad x - z = 3.$$

Adding these two equations together, we get $3x = 10$, or $x = 10/3$. Plugging this back into either equation, we get $z = 1/3$. So a basepoint is $(10/3, 0, 1/3)$ and a parameterization of the line of intersection is

$$l(t) = (10/3, 0, 1/3) + t(-1, 3, 2).$$

16. Consider the parameterized curve

$$\vec{r}(t) = (e^t, t, e^{-t}).$$

(a) (4 points) Find the velocity vector of $\vec{r}$.

Take a derivative:

$$\frac{d\vec{r}}{dt} = (\frac{d(e^t)}{dt}, \frac{d(t)}{dt}, \frac{d(e^{-t})}{dt}) = (e^t, 1, -e^{-t}).$$

(b) (3 points) Is the tangent line to $\vec{r}$ ever parallel to the yz–plane? Be sure to explain your answer.

No. The normal to the yz–plane is the vector $\vec{n} = (1, 0, 0)$, so we'd need

$$0 = \vec{n} \cdot \frac{d\vec{r}}{dt} = (1, 0, 0) \cdot (e^t, 1, -e^{-t}) = e^t.$$

However, exponentials are never zero, so this doesn't happen.

(c) (4 points) Set up, but do not evaluate, the integral to compute to arclength of the section of $\vec{r}$ for $0 \leq t \leq 1$.

The length is given by the integral of the speed:

$$L = \int_0^1 |\frac{\vec{r}}{dt}|dt = \int_0^1 |(e^t, 1, -e^{-t})|dt = \int_0^1 \sqrt{e^{2t} + 1 + e^{-2t}}dt.$$

17. Consider the function $f(x, y) = e^{x^2+y^2-1}$.

(a) (4 points) Sketch the $\{f = 1\}$ level set.

We rewrite the level set equation:

$$\{f = 1\} = \{e^{x^2+y^2-1} = 1\} = \{x^2 + y^2 - 1 = 0\} = \{x^2 + y^2 = 1\}.$$

This is the unit circle centered at the origin.

(b) (4 points) For which values of z does the level set $\{f = z\}$ not contain any points? Be sure to explain your answer.

Notice that exponentials are monotone functions: if you increase the thing in the exponent then you increase the exponential. So f is bounded from below by a lower bound for the exponent, which is $x^2 + y^2 - 1 \geq -1$. Therefore,

$$f(x, y) = e^{x^2+y^2-1} \geq e^{-1}.$$

This tells us that if we choose $z < e^{-1}$ (such as $z = 0$, or $z = 1/10$, or $z = -1$), then the level set $\{f = z\}$ will not contain any points.

18. Consider the function $f(x, y) = xe^y + \cos(xy)$.

(a) (4 points) Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(xe^y + \cos(xy)) = e^y - y\sin(xy), \qquad \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(xe^y + \cos(xy)) = xe^y - x\sin(xy)$$

(b) (4 points) Recall that $(1, 0, \partial f/\partial x)$ and $(0, 1, \partial f/\partial y)$ are two tangent directions for the tangent plane to the graph of f at each of its points. Is this tangent plane ever parallel to the plane $x = y$? Be sure to explain your answer.

We have two tangent vectorsfor the graph, so we can compute a normal vector for the graph:

$$\vec{n}_1 = (1, 0, \frac{\partial f}{\partial x}) \times (0, 1, \frac{\partial f}{\partial y}) = (-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1).$$

Next, we observe that the normal vector to the plane $x = y$ is $\vec{n}_2 = (1, -1, 0)$. We can never have $\vec{n_1} \| \vec{n}_2$, because $\vec{n}_1$ has a nonzero third component, while the $\vec{n}_2$ has 0 for its third component.