

Practice Problems
Math 210
Feb. 7, 2005

These problems are not in any particular order. The exam will be shorter (about or 5 problems).

1. Consider the function $f(x, y) = x^3y - yx^2 + 3xy$.

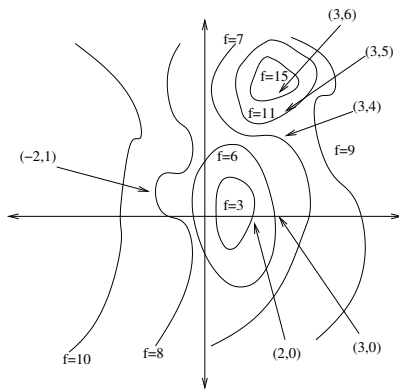
- (a) Write down the equation of the tangent plane to the graph of f at $(0, 1)$.
- (b) At which points (x, y) is the tangent plane horizontal?

2. Consider

$$f(x, y) = \int_x^y e^{t^2} dt.$$

- (a) Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
- (b) Does f have any critical points? Explain your answer.

3. Let some of the level sets of the function f be given by the figure below.



- (a) Estimate $\frac{\partial f}{\partial x}(2, 0)$
- (b) Estimate $\frac{\partial f}{\partial y}(3, 4)$
- (c) In which direction does $\nabla f(-2, 1)$ point?
- (d) Suppose you know $(1, 0)$ is a critical point. Would you guess it's a local maximum, a local minimum, or neither? Explain your answer.

4. Consider $f(x, y) = xe^{x^2+y^2}$ and let $(x_0, y_0) = (1, 1)$.

- (a) Compute $\nabla f(1, 1)$.
- (b) Find the directional derivative $\nabla f \cdot \vec{u}(1, 1)$, where $\vec{u} = (1/2, \sqrt{3}/2)$.
- (c) What is the direction of steepest increase for f , starting at $(1, 1)$. (Be sure to write down a unit vector!)
- (d) Notice $f(1, 1) = e$. What is the equation of the tangent line to the level set $\{f = e\}$, at the point $(1, 1)$?

5. Consider $f(x, y) = \cos x \cos y$.

- (a) Classify all the critical points of f .
- (b) Find the absolute maximum and minimum of f on the square $\pi/4 \leq x \leq 3\pi/4, \pi/4 \leq y \leq 3\pi/4$.

6. Consider the composition $F(t) = f(x(t), y(t))$, where f is a function of the two variables x and y , while x and y are both functions of t .

- (a) Suppose $x(1) = 0, y(1) = 2, x'(1) = 2, y'(1) = -1, \frac{\partial f}{\partial x}(0, 2) = 5$, and $\frac{\partial f}{\partial y}(0, 2) = -3$. Compute $F'(1)$.
- (b) If $x'(0) = 0$ and $y'(0) = 0$, is it true that $F'(0) = 0$? Explain your answer.
- (c) If $F'(3) = 0$, is it true that $x'(3) = 0$ and $y'(3) = 0$? Explain your answer.

7. Compute the slope of the tangent line to the hyperbola $x^2 - y^2 = 1$ at the point $(2, \sqrt{3})$.
8. (a) Evaluate $\int_0^1 \int_0^1 [x^2y + yx^3] dx dy$.
- (b) Evaluate $\int_{x^2+y^2 \leq 1} [e^{x^2+y^2}] dx dy$. (Hint: try polar coordinates.)
- (c) Evaluate $\int_D [xy - x^2y^2] dx dy$, where D is the ice-cream cone shaped region bounded by $y = x - 1$, $y = -x - 1$, and $x^2 + y^2 = 1$. (It might help to draw a picture of D).
9. Find the minimum of $f = y^3 - 3yx^2$ on the ellipse $x^2 + 4y^2 \leq 1$.