Practice Problems

Math 210

Feb. 7, 2005

These problems are not in any particular order. The exam will be shorter (about or 5 problems).

1. Given the following pairs of vectors $\vec{u}$ and $\vec{v}$, find the angle θ between them and compute the cross product $\vec{u} \times \vec{v}$.
   (a) $\vec{u} = (1, 2, 3)$, $\vec{v} = (-2, 1, 0)$
   (b) $\vec{u} = (1, 0, 2)$, $\vec{v} = (2, 1, 0)$
   (c) $\vec{u} = (1, 1, 0)$, $\vec{v} = (1, 0, 1)$
2. Consider the plane Π_1 containing $p = (2, 1, 3)$, with normal vector $\vec{n} = (-1, 2, 0)$.
   (a) Write down the linear equation any point (x, y, z) in this plane must satisfy.
   (b) Find the angle between the plane Π_1 and the plane Π_2 determined by $x - y = 2$.
   (c) Parameterize the line l which is the intersection of Π_1 and Π_2.
   (d) Find the distance between the plane Π_1 and the point $q = (3, 3, 3)$.
3. Consider the vectors $\vec{u} = (1, 2, 1)$ and $\vec{v} = (0, 1, -1)$.
   (a) Explain why all the planes parallel to both $\vec{u}$ and $\vec{v}$ will have the same normal vectors (up to scaling).
   (b) Are these planes parallel to the plane given by $x + y + z = 2$? Explain your answer.
4. Consider the plane curve given by $c(t) = (\cos(t), \sin(2t))$, for $0 \leq t \leq 2\pi$.
   (a) Sketch this curve.
   (b) Set up, but do not evaluate, the integral to compute the arclength of c.
   (c) Notice c is periodic ($c(0) = c(2\pi)$). Is c a simple closed curve? In othre words, are the t parameters 0 and 2π the only times c crosses itself?
   (d) Find the area enclosed by c.
5. (a) Consider the right circular cone C, with vertex at $(0, 0, 0)$, and slope 1. In other words, the cone C is what you get when you rotate the line $y = z$ in the $y - z$ plane about the z-axis. Write C in cylindrical coordinates.
   (b) Write the part of the shell $1 \leq x^2 + y^2 + z^2 \leq 4$ lying in the $x < 0, y > 0, z < 0$ octant in spherical coordinates.
6. Consider the space curve $c(t) = (\cos(t), \sin(t), t)$.
   (a) Is the velocity vector ever tangent to the x-axis?
   (b) Verify the Fundamental Theorem of Calculus by checking
   $$c(2\pi) - c(0) = \int_0^{2\pi} c'(t)dt.$$
   (c) Set up, but do not evaluate, the integral to compute the arclength of c for $0 \leq t \leq 2\pi$.
7. Consider the function $f(x, y) = x^2 - y^2$.
   (a) Sketch the level sets $f = 0$ and $f = 1$.
   (b) Does f have an upper bound? How about a lower bound?
   (c) Compute the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$.
   (d) Is the tangent plane to the graph of f ever parallel to the $x - y$ plane?
8. Explain why the tangent plane to the graph of a function $f(x, y)$ cannot ever be parallel to the $x - z$ or $y - z$ planes, provided f has continuous partial derivatives.