Practice Problems

Math 210

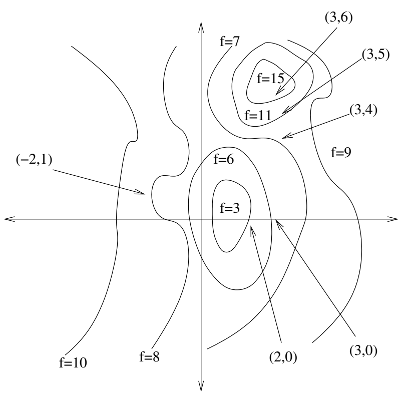
Oct. 14, 2005

These problems are not in any particular order. The exam will be shorter (about or 5 problems).

1. Consider the function $f(x,y) = x^3y - yx^2 + 3xy$.
   (a) Write down the equation of the tangent plane to the graph of f at $(0,1)$.
   (b) At which points (x,y) is the tangent plane horizontal?

2. Consider
$$f(x,y) = \int_x^y e^{t^2}\,dt.$$
   (a) Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
   (b) Does f have any critical points? Explain your answer.

3. Let some of the level sets of the function f be given by the figure below.



   (a) Estimate $\frac{\partial f}{\partial x}(2,0)$
   (b) Estimate $\frac{\partial f}{\partial y}(3,4)$
   (c) In which direction does $\nabla f(-2,1)$ point?
   (d) Suppose you know $(1,0)$ is a critical point. Would you guess it's a local maximum, a local minimum, or neither? Explain your answer.

4. Consider $f(x,y) = xe^{x^2+y^2}$ and let $(x_0,y_0) = (1,1)$.
   (a) Compute $\nabla f(1,1)$.
   (b) Find the directional derivative $\nabla f \cdot \vec{u}(1,1)$, where $\vec{u} = (1/2, \sqrt{3}/2)$.
   (c) What is the direction of steepest increase for f, starting at $(1,1)$. (Be sure to write down a unit vector!)
   (d) Notice $f(1,1) = e$. What is the equation of the tangent line to the level set $\{f = e\}$, at the point $(1,1)$?

5. Consider $f(x,y) = \cos x \cos y$.
   (a) Classify all the critical points of f.
   (b) Find the absolute maximum and minimum of f on the square $\pi/4 \le x \le 3\pi/4$, $\pi/4 \le y \le 3\pi/4$.

6. Consider the composition $F(t) = f(x(t), y(t))$, where f is a function of the two variables x and y, while x and y are both functions of t.
   (a) Suppose $x(1) = 0$, $y(1) = 2$, $x'(1) = 2$, $y'(1) = -1$, $\frac{\partial f}{\partial x}(0,2) = 5$, and $\frac{\partial f}{\partial y}(0,2) = -3$. Compute $F'(1)$.
   (b) If $x'(0) = 0$ and $y'(0) = 0$, is it true that $F'(0) = 0$? Explain your answer.
   (c) If $F'(3) = 0$, is it true that $x'(3) = 0$ and $y'(3) = 0$? Explain your answer.

7. Compute the slope of the tangent line to the hyperbola $x^2 - y^2 = 1$ at the point $(2, \sqrt{3})$.

8. (a) Evaluate $\int_0^1 \int_0^1 [x^2y + yx^3]dxdy$.

   (b) Evaluate $\int_{x^2+y^2 \leq 1} [e^{x^2+y^2}]dxdy$. (Hint: try polar coordiantes.)

   (c) Evaluate $\int_D [xy - x^2y^2]dxdy$, where D is the ice-cream cone shaped region bounded by $y = x-1$, $y = -x-1$, and $x^2 + y^2 = 1$. (It might help to draw a picture of D).

9. Find the minimum of $f = y^3 - 3yx^2$ on the ellipse $x^2 + 4y^2 \leq 1$.

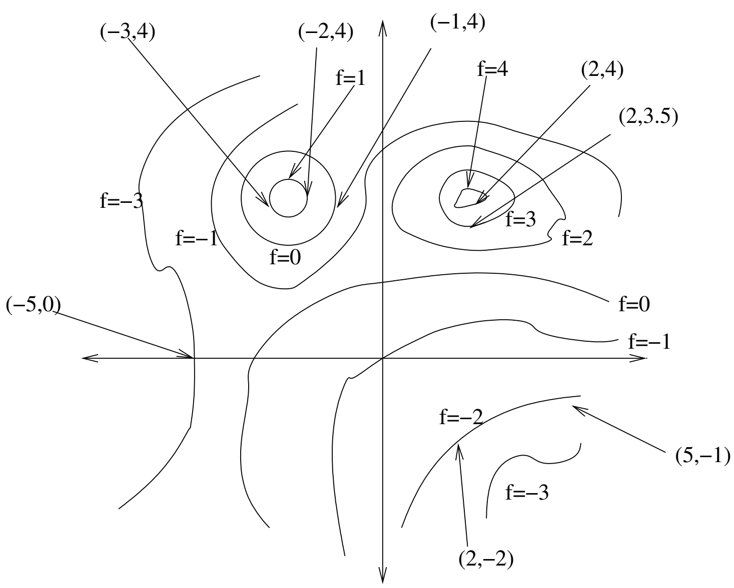
10. Consider the function

$$f(x, y) = \int_{2-y}^{x} \sqrt{1 + t^2}dt.$$

   (a) (5 points) Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

   (b) (5 points) Does f have any critical points? Be sure to explain your answer.

11. Consider the following sketch of level curves of the function $f(x, y)$.



   (a) (3 points) Estimate $\frac{\partial f}{\partial y}(2, 3.5)$.

   (b) (4 points) In which direction does $\nabla f(-5, 0)$ point? Be sure to explain your answer.

   (c) (3 points) If f has a critical point at $(-2.5, 4.5)$, do you expect it to be a local minimum, a local maximum, or a saddle point? Be sure to explain your answer.

12. Consider the function

$$f(x, y) = x^4 + y^4 - 4xy + 1.$$

   (a) (5 points) Verify that the critical points of f are $(0, 0)$, $(1, 1)$, and $(-1, -1)$.

   (b) (5 points) Classify these critical points as local maxima, local minima, or saddle points.

13. (10 points) Find the absolute maximum of $f(x, y) = xy$ on the ellipse $g(x, y) = x^2 + 4y^2 \leq 1$.

14. (a) (5 points) Where D is the square $\{1 \leq x \leq 2, -2 \leq y \leq -1\}$, evaluate

$$\int\int_D [x^2y + yx^3]dA.$$

   (b) (5 points) Set up, but do **not** evaluate, the integral $\int\int_D \sqrt{1 + x^2 + y^2}dA$, where D is the domain bounded by the curves $y = x + 1$ and $x = -y^2$.