

# Kepler's Laws

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## 1 Introduction

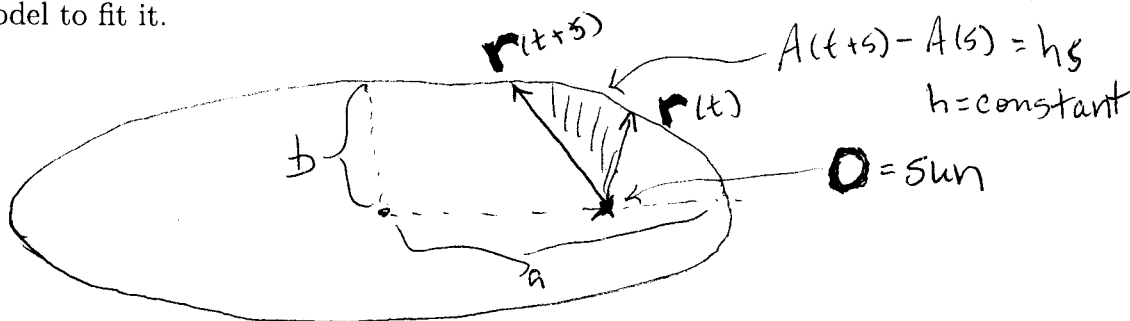
Johannes Kepler was an astronomer in the late 16th and early 17th centuries. His most famous work is his three laws which describe the motion of the planets:

I The orbit of a planet traces out an ellipse in a fixed plane.

II Equal areas are swept out in equal times.

III Let  $T$  be the period of the orbit and  $a$  half the length of the major axis of the ellipse. Then  $T = \tilde{c}a^{3/2}$ , where  $\tilde{c}$  is a constant independent of the planet. The constant  $\tilde{c}$  may depend on the sun.

How did Kepler arrive at these laws? He examined centuries' worth of astronomical data, and came up with a model to fit it.



Amazingly, Isaac Newton (probably around 1680) derived all three of Kepler's laws, starting with some reasonable physical assumptions. Let's make that precise. Newton showed that the only centripetal force which is consistent with Kepler's laws is an inverse square law:

$$|\mathbf{F}| = F = \frac{GmM}{r^2}. \quad (1)$$

Here  $\mathbf{F}$  is the gravitational force between two object of mass  $M$  and  $m$  separated by a distance  $r = |\mathbf{r}|$ , and  $G$  is a universal constant. We can state his three laws of the planets as follows. He also used from the following three laws:

(a) A planet which orbits in a fixed plane containing the sun satisfies Kepler II if and only if the force is centripetal (i.e.  $\mathbf{F}$  and  $\mathbf{r}$  are parallel).

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\*These notes are mostly cribbed from lecture notes by Nick Korevaar, at the University of Utah