Spring 2006 #2

$v(t) = \sqrt{1+t^3}$, want to approx $\int_0^2 v(t)\,dt$ to within $\frac{1}{100}$

using Riemann sums.

we'll need: $v'(t) = \frac{3t^2}{2}(1+t^3)^{-1/2}$ this is positive for $t > 0$

$$v''(t) = 3t(1+t^3)^{-1/2} - \frac{3}{4}t^2(1+t^3)^{-3/2}(3t^2)$$

$$= (1+t^3)^{-3/2}\left[3t(1+t^3) - \frac{9}{4}t^4\right] = \frac{3t + \frac{3}{4}t^4}{(1+t^3)^{3/2}}$$

this is also positive for $t > 0$
(the sign is all we need to know)

$v'' > 0 \Rightarrow v$ is concave up $\Rightarrow$ the graph of v lies above its tangent line.

we can express this as $\underbrace{v(t_j) - v(t_{j-1})}_{\frac{1}{\Delta t}(\text{error at each stage})} \geq \Delta t \cdot v'(t_j)$

$$\geq \Delta t \cdot v'(2)$$ b/c v' is increasing

now plug some things in: $\Delta t = \frac{2-0}{n}$, $v'(2) = \frac{3 \cdot 4}{2}(1+8)^{-1/2}$

$$= 2$$

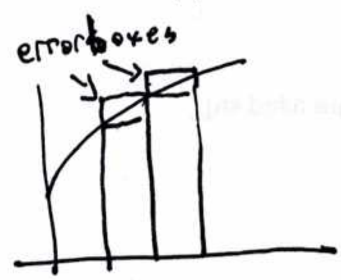
so error at each stage is $\Delta t\,(v(t_j) - v(t_{j-1})) \geq (\Delta t)^2 \cdot v'(2) = \frac{8}{n^2}$

total error is $\sum_{j=1}^{n} \Delta t\,(v(t_j) - v(t_{j-1})) \geq n \cdot (\Delta t)^2 v'(2) = \frac{8}{n}$

want error $< \frac{1}{100}$, so it suffices to have

$$\frac{8}{n} < \frac{1}{100} \iff \boxed{n > 800}$$

pictures: (a)



(b) concave up: