Suppose that a function f define in [0,2] has $\sqrt{4+2x+x^3}$ as an anti-derivative.

a) (3 pts) Find $\int_0^2 f dx$. No approximation is accepted.

$\int_0^2 f(x)dx = \sqrt{4+2x+x^3}\Big|_0^2 = \sqrt{4+4+8} - \sqrt{4} = \sqrt{16} - \sqrt{4} = 4-2 = 2$

b) (2 pts) What is the average value of f in [0,2]?

$\text{avg. val.} = \frac{1}{2}\int_0^2 f(x)dx = \frac{1}{2}\cdot 2 = 1$

A mouse moves back and forth in a straight tunnel. The graph of the mouse's velocity function $v(t)$(feet/seconds) is given in the figure below. The velocity is positive when the mouse moves right and negative when it moves left. Assume that the mouse is at the center of the tunnel when $t = 0$.

a) (5 pts) At which time(s) does the mouse change direction? Explain your answer briefly.

mouse changes direction when $v(t)$ changes sign

$\Rightarrow t \approx 3.2,\ t \approx 4.9,\ t \approx 6.2,\ t \approx 7.9$

b) (5 pts) At which time(s) does the mouse move most rapidly to the right? Explain your answer briefly.

mouse moves most rapidly to right when $v(t)$ is largest $\Rightarrow t \approx 1.5$

c) (5 pts) Using the definite integral, express the farthest distance the mouse reached to the right of the center in the first 10 seconds. You need not calculate the integral. Explain your answer briefly.

mouse gets farthest to right at T = time when $\int_0^T v(t)dt$ is largest, which appears to be $T = 10$

d) (5 pts) Using the the definite integral, express the total distance the mouse traveled in the first 10 second. You need not calculate the integral. Explain your answer briefly.

total displacement is $\int_0^{10} v(t)dt$.