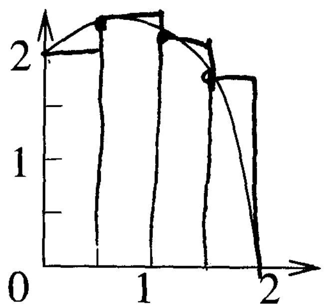
6. Below is a graph of the function $f(x) = -x^2 + x + 2$ on the interval $[0, 2]$.




$f(0) = 2 \qquad f(\tfrac{1}{2}) = -\tfrac{1}{4} + \tfrac{1}{2} + 2 = \tfrac{1}{4} + 2 = \tfrac{9}{4}$

$f(1) = -1 + 1 + 2 = 2$

$f(\tfrac{3}{2}) = -\tfrac{9}{4} + \tfrac{1}{2} + 2 = -\tfrac{7}{4} + 2 = \tfrac{1}{4}$

(a) (5 pts) Approximate the area between the curve and the $x$-axis on the interval $[0, 2]$ by using the left Riemann sum with $n = 4$ subintervals.

$\text{Area} \approx \tfrac{1}{2}\left[f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2})\right] = \tfrac{1}{2}\left[2 + \tfrac{9}{4} + 2 + \tfrac{1}{4}\right]$

$= \tfrac{1}{2}\left[4 + \tfrac{5}{2}\right] = \tfrac{1}{2} \cdot \tfrac{13}{2} = \tfrac{13}{4}$

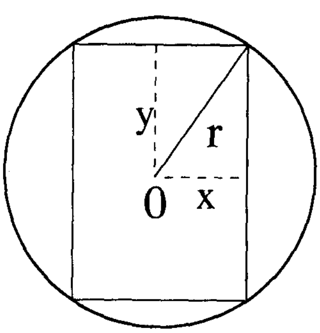
(b) (2 pts) On the graph, draw the rectangles which correspond to the calculation in (a).

(c) (3 pts) Use the Fundamental Theorem of Calculus to calculate the area of the region exactly. Show your work.

$\text{Area} = \int_0^2 -x^2 + x + 2 \, dx = \left[-\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + 2x\right]_0^2$

$= -\tfrac{8}{3} + 2 + 4 = -\tfrac{8}{3} + 6 = \tfrac{10}{3}$

7. (10 pts) What are the dimensions of a rectangle that is inscribed to a circle of a constant radius $r$ and that maximizes the area among all such inscribed rectangles? One may proceed as follows. First, find a relation among $x, y, r$. Next, find the formula expressing the area of such a rectangle in $x, y$. Then eliminate one of the variables and use the first derivative test. Show your work.




note $x^2 + y^2 = r^2 \Rightarrow y = \sqrt{r^2 - x^2}$

$\text{Area} = 4xy = 4x\sqrt{r^2 - x^2} = A(x)$

$A'(x) = 4\sqrt{r^2 - x^2} + 4x\left(\tfrac{1}{2}(r^2 - x^2)^{-1/2}(-2x)\right)$

$= 4\sqrt{r^2 - x^2} - \dfrac{4x^2}{\sqrt{r^2 - x^2}} = 0$

$\Leftrightarrow 4\sqrt{r^2 - x^2} = \dfrac{4x^2}{\sqrt{r^2 - x^2}} \Leftrightarrow 4r^2 - 4x^2 = 4x^2$

$\Leftrightarrow 4r^2 = 8x^2$

$\Leftrightarrow x = r/\sqrt{2}$

Candidates for max: $x = r/\sqrt{2}$, $\underbrace{x = 0, \; x = r}_{\text{end points}}$.

note