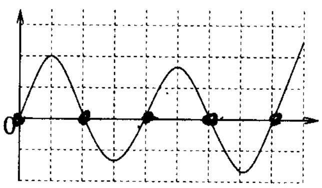
4. The figure below is the graph of the **derivative** $f'(x)$ of a function $f(x)$. The unit on the x-axis is 1 and on the y-axis is 10.



crit. pnts where $f' = 0$ at $x = 0, 2, 4, 6, 8$
can't tell if $x=0$ is an extremum
b/c don't know f' for $x<0$

a) (5 pts) At which x values (roughly) does f have local extrema? Describe the nature of the extrema (local max or min) and explain your answers briefly.

extrema: $x = 2, 4, 6, 8$ max: $x = 2, 6$ b/c $f' > 0$ to left, $f' < 0$ to right
min: $x = 4, 8$ b/c $f' < 0$ to left, $f' > 0$ to right

b) (4 pts) In which interval(s) is f concave-up. Explain your answers briefly.

f concave up $\iff f'' > 0 \iff f'$ increasing
$\iff x \in (0,1)$, $x \in (3,5)$, $x \in (7,9)$

c) (3 pts) At which point(s) does f have inflection points? Explain your answers briefly.

inflection pnts $\iff f''$ changes sign $\iff$ local max or min for f'
$\iff x = 1, 3, 5, 7$

d) (4 pts) Assuming that $f(0) = 0$, is it true or false that the global max of f in [0,9] is likely to be a positive number? Explain your answer briefly.

true: not $f' > 0$ on $0 < x < 1 \Rightarrow f$ must increase from zero

e) (4 pts) Assuming that $f(0) = 0$, is it true or false that the global min of f in [0,9] is likely to be a positive number? Explain your answer briefly.

the global min occurs either at $x=4$ or $x=8$, depending on the areas of the bumps. It appears that the last bump is ~~biggest~~ larger than the 2nd and 3rd bumps, but not ~~as~~ big as the 1st bump. Meanwhile, the 2nd bump is smallest. In conclusion, yes it's likely that the global ~~min~~ min (which is a difference in areas of these bumps) is positive

5. (5 pts) A spherical snow ball is melting. It radius decreases at a constant rate of 3 cm per minute from an initial value of 100 cm. How fast is the volume decreasing 10 minutes later? The volume is given by $V = 4/3\pi r^3$.

$\frac{dr}{dt} = -3$, $V = \frac{4}{3}\pi r^3$ $r(0) = 100$, $r(t) = 100 - 3t$
$r(10) = 100 - 30 = 70$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 \frac{dr}{dt}$$

$$\left.\frac{dV}{dt}\right|_{10} = 4\pi \cdot 70^2 (-3) \simeq -184726 \ \text{cm}^3/\text{min}$$