

Name: Solutions

Section: _____

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. No explanation is necessary.

(a) (3 pts) A local maximum of a function is always a critical point. (a) T **F**

(b) (3 pts) If $\int_a^b f(x)dx = 0$ implies that f is the constant zero function. (b) T **F**

(c) (3 pts) According to L'Hopital's rule, $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} = -\frac{1}{6}$. (c) **T** F

$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} = \lim_{x \rightarrow 0} \frac{-\sin x}{6x} = \lim_{x \rightarrow 0} \frac{-\cos x}{6} = -\frac{1}{6}$

(d) (3 pts) If $f''(x) = 0$ at $x = 0$, then the graph of f changes concavity at $x = 0$. (d) T **F**

(e) (3 pts) If a function is not continuous at a point, then the function is not defined at that point.

(e) T **F**

2. (7 pts) An object is traveling along a line. Its velocity is described by the function $v(t) = \sqrt{1+t^3}$, where its units are meters per seconds. In order to approximate the distance traveled by the objects in the first 2 seconds within 0.01 meters, how often does one have to record the velocity of the object?

see attached: solution will not fit in this space

3. Let $f(x) = \sqrt{x+1}$. $f'(x) = \frac{1}{2\sqrt{x+1}}$ $f'(0) = \frac{1}{2} = \text{slope of tangent line}$

(a) (5 pts) Find the equation of the tangent line to the graph of $f(x)$ at $x = 0$.

eqn of tangent line: $y - 1 = \frac{1}{2}(x - 0)$ or $y = \frac{1}{2}x + 1$

(b) (3 pts) Use the tangent line approximation to estimate $\sqrt{0.9}$.

$$\sqrt{0.9} = f(-.1) = f(-\frac{1}{10}) \approx \frac{1}{2}(-\frac{1}{10}) + 1 = \frac{19}{20} = 0.95$$