

2. [7 pts.] The differentiable function g has the following properties:
 $g(2) = 5$, $g'(2) = 3$, $g(0) = -2$, $g'(0) = 4$. Show your work below.

(a) If $f(x) = \ln(g(x))$, find $f'(2)$.

$$f'(2) = \left. \frac{d}{dx} (\ln g(x)) \right|_{x=2} = \frac{g'(2)}{g(2)} = \frac{3}{5}$$

Final answer to (a):

$$f'(2) = \frac{3}{5}$$

(b) If $h(x) = \frac{g(x)}{x^2+1}$, find $h'(0)$.

$$\begin{aligned} h'(0) &= \left. \frac{d}{dx} \left[\frac{g(x)}{x^2+1} \right] \right|_{x=0} = \left. \frac{d}{dx} [g(x)(x^2+1)^{-1}] \right|_{x=0} \\ &= \left. [g'(x)(x^2+1)^{-1} - 2xg(x)(x^2+1)^{-2}] \right|_{x=0} \\ &= \left. \left[\frac{(x^2+1)g'(x) - 2x \cdot g(x)}{(x^2+1)^2} \right] \right|_{x=0} = \frac{1 \cdot 4 - 0 \cdot (-2)}{1^2} = \frac{1}{4} \end{aligned}$$

Final answer to (b):

$$h'(0) = \frac{1}{4}$$