(10 pts) A rectangular storage container with an open top is to have a volume of 10 ft^3. The length of its base is twice the width of the base. Material for the base costs \$10 per square foot. Material for the sides costs \$6 per square foot. Find the cost of materials for the cheapest such container.

$l = 2w \qquad V = w \cdot 2w \cdot h = 10 \quad \Rightarrow$ ~~$5w^2h = 10 \Rightarrow h =$~~

$\Rightarrow w^2 h = 5 \Rightarrow h = 5/w^2$

Area of base $= 2w^2$, area of sides $= 2lh + 2wh = 4wh + 2wh$

$= 6wh = 30/w$

$\therefore$ total cost is $C = 20w^2 + 180/w$

$\frac{dC}{dw} = 40w - \frac{180}{w^2} = 0 \quad \Rightarrow \quad 40w = \frac{180}{w^2} \Rightarrow w^3 = \frac{18}{4} = 9/2$

$\Rightarrow w = (9/2)^{1/3}$

$\left.\frac{d^2C}{dw^2}\right|_{(9/2)^{1/3}} = \left.\left(40 + \frac{360}{w^3}\right)\right|_{(9/2)^{1/3}} = 40 + \frac{720}{9} > 0 \quad \Rightarrow \quad w = (9/2)^{1/3}$ is min.

so the cost is $C = 20(9/2)^{2/3} + 180(9/2)^{-1/3}$

(10 pts) (a) Use the tangent line approximation for $f(x) = \sqrt{x}$ near $x = 16$ to approximate $\sqrt{16.2}$.

note $f(16) = 4$, $\quad f'(x) = \frac{1}{2}x^{-1/2}$, $\quad f'(16) = 1/8$

$\therefore \sqrt{16.2} \simeq f(16) + (16.2 - 16) \cdot f'(16) = 4 + \frac{1}{5} \cdot \frac{1}{8}$

$= 4 + \frac{1}{40} = \frac{161}{40}$

(b) The approximation you found in (a) should be slightly larger than the actual value of $\sqrt{16.2}$. Show how you can use the **The Racetrack Principle** to predict that this is the case. (Hint: Let $f(x) = \sqrt{x}$ and let $g(x)$ be the local linearization of $\sqrt{x}$. You want to show that $g(x) > f(x)$ when $x = 16.2$.)

let $f = \sqrt{x}$, $g = 4 + (x-4) \cdot \frac{1}{8}$

then $f' = \frac{1}{2\sqrt{x}}$, $g' = \frac{1}{8}$ so for $x \geq 16$, $f'(x) \leq g'(x)$

also, $f(16) = 4 = g(4)$

So by the racetrack principle $f(x) < g(x)$ for $x > 16$.