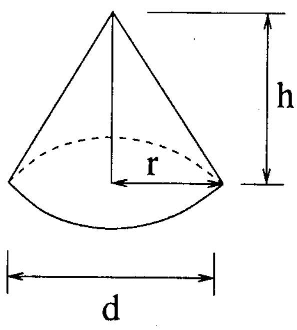
(10 pts) Sand is pouring from a pipe at the rate of 16 cubic feet per second. If the falling sand forms a conical pile on the ground whose height is always 1/4 the diameter of the base, how fast is the height of the pile increasing when the pile is 4 feet high. (Recall that the volume of a circular cone with base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.)



$h = \frac{2r}{4} = \frac{r}{2}$ or $r = 2h$

$\Rightarrow V = \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi h^3$

$\Rightarrow 16 = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi h^3\right) = 4\pi h^2 \cdot \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2}$

evalute at $h = 4 \Rightarrow \frac{dh}{dt} = \frac{1}{4\pi}$

(10 pts) Evaluate the following limits. Justify your answers (for example, use L'Hopital's Rule when appropriate or use properties of limits).

(a) $\lim_{x\to 1} \frac{\ln(x)}{x^2-1} = \lim_{x\to 1}\left[\frac{1/x}{2x}\right] = \frac{1}{2}$

b/c $\ln x \to 0$ and $x^2-1 \to 0$ can use L'Hospital's rule.

(b) $\lim_{x\to 1^+} \frac{x}{\ln(x)}$ DNE.

the numerator is near 1 but the denominator is close to zero, so the ratios get huge as $x \to 1^+$

$\Rightarrow \lim(\text{—})$ DNE