

2. (10 pts) Multiple choice: Circle all the correct answers. Some problems may have more than one correct answer.

(a) Suppose $h(x)$ is differentiable on the closed interval $[0,5]$ and has a local maximum at $x = 3$. Which of the following must be true?

(i) $h'(3)$ is equal to zero or $h(3)$ is an end point.

(ii) h has a critical point at $x = 3$.

(iii) $h''(3)$ is positive.

(iv) $h''(3)$ is negative.

(v) $h'(2) > 0$ and $h'(4) < 0$.

(b) Suppose f is an increasing function that is differentiable for all x . Assume that $f(10) = 5$ and $f'(10) = 8$.

(i) $f^{-1}(5) = 10$.

(ii) $(f^{-1})'(5) = \frac{1}{8}$.

(iii) $(f^{-1})'(5) = 8$.

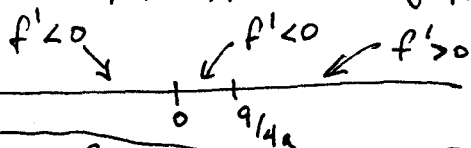
(iv) (f^{-1}) is a decreasing function.

(v) (f^{-1}) is an increasing function.

3. (15 pts) For the function $f(x) = ax^4 - 3x^3$ with constant $a > 0$, use the techniques of calculus to answer the following. Show your work with proper justification for your answers.

(a) Determine all critical points of f . Classify each as a local maximum, local minimum or neither.

$$f'(x) = 4ax^3 - 9x^2 = x^2(4ax - 9) = 0 \Leftrightarrow x = 0, x = 9/4a$$



for $x \gg 0$, $f'(x) \approx 4ax^3 \Rightarrow f'(x) > 0$
for x large, post

so $x = 9/4a$ is a local min
and $x = 0$ is neither a max
nor a min

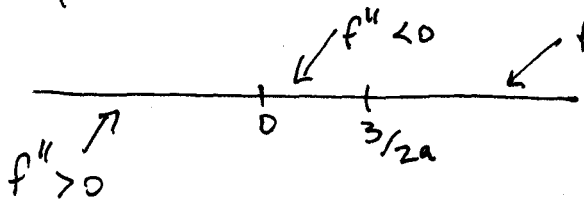
also, by same reasoning, $f'(x) < 0$ for x large, neg.
for $x \ll 0$, $f'(x) \approx -9x^2 \Rightarrow f'(x) < 0$ for $|x|$ small

(b) Determine any global maxima or minima (if any).

None. the dominant term is a cubic, which can be large and neg, as well as large and pos.

(c) Determine all (if any) inflection points.

$$f'' = 12ax^2 - 18x = 6x(2ax - 3) = 0 \Leftrightarrow x = 0, x = 3/2a$$



$x = 0$ is the only inflection point.