

8. [10 points] Consider the continuous function

$$f(x) = x^3 + 3x^2 - 24x - 4$$

on the closed interval $-4 \leq x \leq 4$.

(a) At what values of x does f have a local minimum? A local maximum? Please show your work and give exact answers in the box provided.

$$f' = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x+4)(x-2)$$

$$\therefore f' = 0 \Leftrightarrow x = -4, x = 2$$

$$f'' = 6x + 6 \quad f''(-4) = -24 + 6 = -18 < 0 \Rightarrow -4 \text{ is a local max,}$$
$$f''(2) = 12 + 6 = 18 > 0 \Rightarrow 2 \text{ is a local min}$$

Final answer to (a):

$x = -4$ is a loc. max

$x = 2$ is a loc. min.

(b) Does f achieve a global maximum and a global minimum on this interval? State why or why not. If they do exist, what are their values?

yes, f is continuous, so it must achieve its ~~to~~ global max and min.

test values $f(-2) = -8 + 12 + 48 - 4 = 48$

$f(4) = 64 + 48 - 96 - 4 = 12 \leftarrow$ global min

$f(-4) = -64 + 48 + 96 - 4 = 76 \leftarrow$ global max

$$\begin{array}{r} 64 \\ 48 \\ \hline 112 \end{array} \quad \begin{array}{r} 96 \\ 48 \\ \hline 144 \\ -68 \\ \hline 76 \end{array}$$