7. [10 points] Please note that the formula for the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$, and the formula for the surface area of a sphere of radius r is $A = 4\pi r^2$. A clown is filling a balloon with helium from a tank that emits helium at a rate of $250\text{cm}^3/\text{sec}$.

Please answer the following questions, specifying the **units of your answers**. Show your work, and put your final answer in the box provided.

(a) How fast is the radius of the balloon changing when the radius is 5cm?

$$250 = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 \cdot \frac{dr}{dt} \Rightarrow \left.\frac{dr}{dt}\right|_{r=5} = \left.\frac{250}{4\pi r^2}\right|_{r=5} = \frac{250}{4\cdot\pi\cdot 5^2} = \frac{2.5}{\pi}$$

Final answer to (a): $\frac{2.5}{\pi}$ cm/s

$A(5) = 4\cdot\pi\cdot 25 = 100\pi$

$\frac{A}{2\pi} = 25$

(b) How fast is the surface area of the balloon changing when the radius of the balloon is 5cm?

$$V = \frac{1}{3}A\cdot r \quad \text{or} \quad A = 4\pi r^2 \Rightarrow r = \sqrt{\frac{A}{4\pi}} \Rightarrow V = \frac{4}{3}\pi\left(\frac{A}{4\pi}\right)^{3/2}$$

$$\Rightarrow 250 = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi\left(\frac{A}{4\pi}\right)^{3/2}\right) = \frac{4}{3}\pi\left(\frac{A}{4\pi}\right)^{1/2}\cdot\frac{3}{2}\cdot\frac{dA}{dt}$$

$$250 = 2\pi\cdot 5\cdot\frac{dA}{dt} \Rightarrow \frac{dA}{dt} = \frac{250}{10\pi} = \frac{25}{\pi}$$

Final answer to (b): $25/\pi$ cm^2/s

(c) After 20 seconds, the balloon escapes the clown's grip. Helium blows out of the balloon at a rate of $350\text{cm}^3/\text{sec}$. How fast is the radius changing when the radius of the balloon is 10cm?

at $r = 10$ cm:
$$-350 = \frac{dV}{dt} = 4\pi r^2\cdot\frac{dr}{dt} = \frac{dr}{dt} = -\frac{350}{4\pi r^2} = -\frac{350}{400\pi}$$
$$= -\frac{7}{8\pi}$$

Final answer to (c): $-\frac{7}{8\pi}$ cm/s