

4. [12 points] Please **CIRCLE** True or False for each statement. If the statement is False, then give an example contradicting the statement; if the statement is true, then state a relevant theorem that guarantees it is true.

(a) If  $f$  is continuous for  $0 < x < 1$  and achieves a local maximum at  $x_0 = \frac{1}{2}$ , then  $f'(\frac{1}{2}) = 0$ . TRUE FALSE

JUSTIFICATION:

$f$  could look like



(b) If  $f$  is differentiable for  $-2 < x < 2$  and  $f(-1) = f(1)$ , then there exists a value  $c$  with  $-1 < c < 1$  such that  $f'(c) = 0$ . TRUE FALSE

JUSTIFICATION:

mean value theorem.

(c) If  $f'(x) > 0$  for all  $x$ , then  $f$  is an increasing function. TRUE FALSE

JUSTIFICATION:

one of the properties of  $f'$

(d) If a differentiable function  $f$  is defined at  $c$ , and  $c$  is a critical point of  $f$ , then  $f$  has either a local maximum or a local minimum at  $c$ . TRUE FALSE

JUSTIFICATION:

e.g.  $f(x) = x^3$ ,  $c = 0$

(e) If  $f$  is a differentiable function and  $f(0) < f(1)$ , then there is some  $0 < c < 1$  with  $f'(c) > 0$ . TRUE FALSE

JUSTIFICATION:

mean value theorem.