3. [12 points] Suppose y is defined implicitly by the relation $\sin(2x) - \cos(3y^2) = 1$.

(a) Find $\frac{dy}{dx}$. Show your work, and put your final answer in the box below.

$1 = \sin 2x - \cos(3y^2)$

$0 = 2\cos(2x) + \sin(3y^2) \cdot 6y \cdot y'$

$-2\cos(2x) = \sin(3y^2) \cdot 6y \cdot y'$

$y' = -\dfrac{2\cos(2x)}{6y\sin(3y^2)}$

Final answer to (a):

$\frac{dy}{dx} = -\dfrac{\cos(2x)}{3y\sin(3y^2)}$

(b) What is the equation of the tangent line to the curve $\sin(2x) - \cos(3y^2) = 1$ at the point $(\frac{\pi}{4}, \sqrt{\frac{\pi}{6}})$? Sh your work, and put your final answer in the box below.

slope is $y' = -\dfrac{\cos(\pi/4)}{3\sqrt{\frac{\pi}{6}}\sin(3\cdot(\pi/6))} = -\dfrac{1/\sqrt{2}}{3\sqrt{\frac{\pi}{6}}\sin(\pi/2)} = -\dfrac{1}{3\sqrt{\pi/3}}$

$= -\dfrac{1}{\sqrt{3\pi}}$

Final answer to (b):

$y - \sqrt{\frac{\pi}{6}} = -\frac{1}{\sqrt{3\pi}}(x - \pi/4)$

(c) Are there points on the curve $\sin(2x) - \cos(3y^2) = 1$ where the tangent lines are vertical? Explain yo answer.

yes. when $3y^2 \to n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

then the denominator to $y' \to 0$

$\Rightarrow$ slope $\to \pm\infty$

$\Rightarrow$ tangent line approaches vertical.