

2. [10 points] Suppose that $f(x) = xe^x$. Show your work on the following problems, and write your final answer in the box provided.

(a) Determine the value of $f^{-1}(e)$?

$$f(x) = e = xe^x \Leftrightarrow x = 1$$

Final answer to (a):

$$f^{-1}(e) = 1$$

(b) Determine the value of $(f^{-1})'(e)$?

$$\begin{aligned} \text{let } g(x) &= f^{-1}(x) \\ \text{then } g'(x) &= \frac{1}{f'(g(x))} \Rightarrow g'(e) = \frac{1}{f'(1)} = \frac{1}{(1+1)e^1} = \frac{1}{2e} \end{aligned}$$

$$f'(x) = e^x + xe^x = (x+1)e^x$$

Final answer to (b):

$$(f^{-1})'(e) = \frac{1}{2e}$$