

1. [10 points] Suppose that a function f satisfies $f(1) = 3$ and its DERIVATIVE is

$$\frac{d}{dx} (f(x)) = e^{-x^2}.$$

In (a) and (b), please show your work, and write your final answer in the box provided.

(a) Find the local linearization of $f(x)$ near $x = 1$.

$$\begin{aligned} f(x) &\approx f(1) + (x-1) f'(1) \\ &= 3 + (x-1) e^{-1} = \frac{x}{e} + 3 - \frac{1}{e} \end{aligned}$$

Final answer to (a):

$$f \approx \frac{x}{e} + 3 - \frac{1}{e}$$

(b) Estimate the value of $f(1.5)$.

$$f(1.5) \approx \frac{1.5}{e} + 3 - \frac{1}{e} = \frac{3}{2} \cdot \frac{1}{e} + 3 - \frac{1}{e}$$

Final answer to (b):

$$f(1.5) \approx 3 + \frac{1}{2e}$$

(c) Is your answer in (b) an under- or an over-estimate? Please give an argument supporting your answer.

$$\text{note } f' > 0 \text{ and } f'' = -2xe^{-x^2} < 0$$

$\Rightarrow f$ is concave down
 $\Rightarrow f$ lies under its tangent line
 \Rightarrow the linearization is an under estimate.