

9. A function $f(x) = x^3 - 6x + 1$ is given in $[-2, 3]$.

a) (7 points) Find the local extrema (maxima and minima) of f using the first derivative test. Clearly indicate where these local extrema occur and what values they are.

$$f' = 3x^2 - 6 = 3(x^2 - 2) = 3(x - \sqrt{2})(x + \sqrt{2}) = 0$$

interior crit. pts at $x = \sqrt{2}, -\sqrt{2}$

$$f'' = 6x, \quad f''(\sqrt{2}) = 6\sqrt{2} > 0 \Rightarrow x = \sqrt{2} \text{ is local min}$$

$$f''(-\sqrt{2}) = -6\sqrt{2} < 0 \Rightarrow x = -\sqrt{2} \text{ is local max}$$

b) (3 points) Find the global (absolute) extrema of f . Clearly indicate where these global extrema occur and what values they are.

$$f(\sqrt{2}) = 2\sqrt{2} - 6\sqrt{2} + 1 = -4\sqrt{2} + 1 \leftarrow \text{global min}$$

$$f(-\sqrt{2}) = -2\sqrt{2} + 6\sqrt{2} + 1 = 4\sqrt{2} + 1$$

$$f(-2) = -8 + 12 + 1 = 5$$

$$f(3) = 27 - 18 + 1 = 10 \leftarrow \text{global max}$$

c) (3 points) Find the inflection points if any.

inflection pt at $x=0$ b/c $f'' < 0$ for $x < 0$
 $f'' > 0$ for $x > 0$

d) (5 points) Sketch the graph of f . Make sure all pertinent pieces of information such as local and global extrema, y intercept, inflection points, concavity etc to be indicated.

