Suppose two functions $f(x) = e^x$ and $g(x) = x + 1$ are given for all values of x.

(a) (4 points) Explain graphically why $f(x) \geq g(x)$ for all x.

g is the tangent line to f @ $x=0$ and f is concave up.

(b) (4 points) Prove $f(x) \geq g(x)$ for all x using the racetrack principle.

note $g(0) = 1 = f(0)$, $g' = 1$, $f' = e^x$

if $x > 0$ then $f' > g' \Rightarrow f > 0$ if $x < 0$ then $f' < g' \Rightarrow f < g$

Below is the graph of $f(x)$, the derivative of $F(x)$ (that is, $F'(x) = f(x)$). Answer the following:

(a) (2 points) What are the interval(s) on which $F(x)$ is increasing?

(b) (2 points) What are the interval(s) on which $F(x)$ is concave down?

(c) (2 points) What value(s) of x will give a local maximum for $F(x)$?

(d) (2 points) Suppose $F(0) = 2$. Determine the value of $F(4)$.

The table gives values for functions f and g, and their derivatives. Determine the given derivatives and write down the formulas used.

x	0	1	2
$f(x)$	5	-1	4
$g(x)$	2	2	0
$f'(x)$	-2	2	3
$g'(x)$	5	6	0.5

(a) (2 points) If $K(x) = \dfrac{f(x)}{g(x)}$ then $K'(0) = \dfrac{f'(0)g(0) - g'(0)f(0)}{g^2(0)} = \dfrac{-4-25}{4} = -\dfrac{29}{4}$?

$K' = (f \cdot g^{-1})' = f'g^{-1} + f(-g^{-2} \cdot g') = \dfrac{f'g - g'f}{g^2}$

(b) (2 points) If $H(x) = f(g(x))$ then $H'(1) = f'(g(1))g'(1) = 3 \cdot 6 = 18$?

(c) (2 points) If $F(x) = e^{f(x)}$ then $F'(1) = f'(1) \cdot e^{f(1)} = 2 \cdot e^{-1} = 2/e$?

(d) (2 points) If $G(x) = (f(x))^2$ then $G'(1) = 2f(1) \cdot f'(1) = -2 \cdot 2 = -4$?

(c) (2 points) If $K(x) = \sin(g(x))$ then $K'(2) = \cos(g(2)) \cdot g'(2)$?

$= \cos(0) \cdot 1/2 = 1/2$