3. (10 points) The volume of a certain tree is given by $V = \frac{1}{12\pi}C^2h$, where C is the circumference of the tree at the ground level and h is the height of the gree. If C is 5 feet and growing at the rate of 0.2 feet per year, and if h is 22 feet and is growing at 4 feet per year, find the rate of growth of the volume V.

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{1}{12\pi} C^2 h\right] = \frac{1}{12\pi}\left[2C\frac{dC}{dt}h + C^2\frac{dh}{dt}\right]$$

$$= \frac{1}{12\pi}\left[2\cdot 5\cdot \frac{1}{5}\cdot 22 + 25\cdot 4\right] = \frac{1}{12\pi}\left[44+100\right] = \frac{144}{12\pi} = \frac{12}{\pi}$$

4. Let g be a function such that $g(2) = 4$ and whose derivative is known to be $g'(x) = \sqrt{x^2+1}$.

(a) (5 points) Use a linear approximation to estimate the value of $g(1.95)$. Show your work.

$$g(1.95) \simeq g(2) + (1.95-2)g'(2) = 4 - .05\left(\sqrt{2^2+1}\right) = 4 - .05\cdot\sqrt{5}$$
$$\simeq 3.89$$

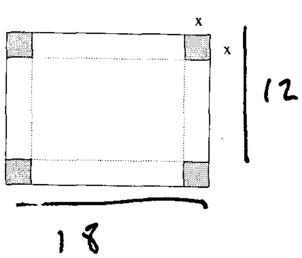
(b) (5 points) Do you think your estimate in part (a) is an overestimate or an underestimate? Explain.

note $g' > 0$, $g'' = x/\sqrt{x^2+1} > 0 \Rightarrow g$ is concave up

$\therefore$ the graph of g is above its tangent line

$\Rightarrow$ the lin. approx is ~~an~~ underestimate

5. (10 points) An open top rectangular box is to be made from a 12×18 inch piece of cardboard by cutting a square of side x from each corner and folding up the sides along the dotted edges (See figure below). Find x at which the resulting box has the (global) maximum volume.



$6\overline{)216}$ 36

$0 \le x \le 6$ height $= x$

width $= 12-2x$ depth $= 18-2x$

$$V = x(12-2x)(18-2x) = 4x(6-x)(9-x) =$$
$$= 4x(x^2-15x+54) = 4x^3 - 60x^2 + 216x$$

$$V' = 12x^2 - 120x + 216 = 6(2x^2-20x+36)$$
$$= 12(x^2-10x+18) = 0 \qquad x = \frac{10 \pm \sqrt{100-72}}{2} = \frac{10\pm\sqrt{28}}{2}$$

take $x = \frac{10-\sqrt{28}}{2} = 5-\sqrt{7} \simeq 2.35$

this is the only interior crit pnt.

note $V(5-\sqrt{7}) > 0$

and $V(0) = V(6) = 0$

$\therefore$ V is maximized at $x = 5-\sqrt{7}$