7. (15 pts) (a) (5 PTS) Write the limit definition for the derivative of $f(x) = \frac{1}{x} + 3$.

$$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{x\to 0}\left[\frac{\left(\frac{1}{x+h}+3\right)-\left(\frac{1}{x}+3\right)}{h}\right]$$

(b) (10 PTS) Find $f'(x)$ **using algebra and the limit definition** in (a). No credit will be given f simply using the formulas for writing down the derivative.

$$f'(x) = \lim_{h\to 0}\left[\frac{1}{h}\left(\frac{1}{x+h}-\frac{1}{x}\right)\right] = \lim_{h\to 0}\left[\frac{1}{h}\left(\frac{x-(x+h)}{x(x+h)}\right)\right] = \lim_{h\to 0}\left[\frac{1}{h}\left(\frac{-h}{x^2+xh}\right)\right]$$

$$= \lim_{h\to 0}\left[\frac{-1}{x^2+hx}\right] = -\frac{1}{x^2}$$

8. (10 pts) A toy rocket is launched upward from the roof of a building 112 feet high. It rises and then fal back; its height above the ground t seconds after it is thrown is $y = -16t^2 + 96t + 112$ feet, until it hi the ground.

(a) (4 PTS) Find the average velocity of the rocket during the first 2 seconds? Give units.

$$\text{avg. velocity} = \frac{y(2)-y(0)}{2-0} = \frac{240-112}{2} = 64 \text{ ft/sec.}$$

(b) (6 PTS) When does the rocket hit the ground and how fast is it going at that time? Give units.

rocket hits ground at $0 = y(t) = -16t^2+96t+112$

$$= -16(t^2-6t-7) = -16(t-7)(t+1)$$

$$\Rightarrow t = 7$$

$$y'(t) = -32t + 96$$

$$y'(7) = -32\cdot 7 + 96 = -128 \text{ ft/sec.}$$