

10. (10 pts) A particle moves in such a way that its position along the y -axis is given by $y(t) = 3t^2 - 8\frac{1}{\sqrt{t}}$ over the time interval $1 \leq t \leq 10$.

(a) Find $\frac{dy}{dt}$: $y' = 6t + 8 \cdot \frac{1}{2} \cdot t^{-3/2}$
 $= 6t + 4t^{-3/2}$

- (b) What is the velocity at $t = 4$?

$$\text{velocity} = y'(4) = 6 \cdot 4 + 4(4)^{-3/2} = 24 + \frac{4}{2^3} = 24 + \frac{1}{2} = 24\frac{1}{2}$$

- (c) What is the acceleration at $t = 4$? $y'' = 6 - 6t^{-5/2}$

$$\text{acceleration} = y''(4) = 6 - \frac{6}{2^5} = 6 - \frac{6}{32} = \frac{192}{32} \approx 5.81$$

- (d) When is the velocity decreasing (find all times t)?

$$\text{velocity is decreasing} \Leftrightarrow y'' < 0 \Leftrightarrow 6 < 6t^{-5/2} \Leftrightarrow t < 1$$

11. (5 pts) The table below gives the values of a function $f(x)$ at various x .

x	0	1	2	3	4
$f(x)$	0.5	0.580	0.912	1.691	3.218

- a) What is the average rate of change in the interval $[1, 3]$?

$$\text{avg. rate of change} = \frac{f(3) - f(1)}{3 - 1} = \frac{1.691 - 0.580}{2} = .5555$$

- b) What is possibly the most appropriate approximation of $f'(2)$? Justify your answer

$$f'(2) \approx \frac{f(3) - f(1)}{3 - 1} = .779$$

$$\text{or } f'(2) \approx \frac{f(2) - f(1)}{2 - 1} = .332$$

or you could average these two to get

$$f'(2) \approx \frac{f(3) - f(1)}{3 - 1} = .5555$$