3. (10 pts) (a) Use the definition of the derivative and algebra to evaluate the derivative of the function $f(x) = 1/x$ at $x = 3$.

$$f'(3) = \lim_{h\to 0}\left[\frac{f(3+h)-f(3)}{h}\right] = \lim_{h\to 0}\left[\frac{\frac{1}{3+h}-\frac{1}{3}}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\frac{3-(3+h)}{9+3h}}{h}\right] = \lim_{h\to 0}\left[\frac{1}{h}\cdot\frac{-h}{9+3h}\right] = \lim_{h\to 0}\left[\frac{-1}{9+3h}\right] =$$

$$= \boxed{-\frac{1}{9}}$$

(b) Find the equation of the tangent line to $y = 1/x$ at $x = 3$.

$f(3) = 1/3$, slope $= f'(3) = -1/9$

so $y - \frac{1}{3} = -\frac{1}{9}(x-3)$

$500(-x^2+2x-3)$

(10 pts) Let $f'(x) = 500(x-1)(3-x)$ (NOTE: This is the derivative of f).

(a) Determine the intervals on which f is increasing. f is increasing when $f'>0$

could have $x-1>0$ and $3-x>0$ i.e. $x>1$ and $x<3$

i.e $1<x<3$

or could have $x-1<0$ and $3-x<0$ i.e. $x<1$ and $x>3$

this last one is impossible,

so yout only get $1<x<3$

(b) Find $f''(x)$.

$f'' = 500(3-x) - 500(x-1) = \boxed{-1000x+1000}$

(c) Determine the inflection points of $f(x)$.

inflection pnts $\iff f''(x)=0 \iff -1000x+1000=0$

$\iff \boxed{x=1}$