

(10 pts) $f(x) = \begin{cases} 8-x & \text{for } x < 4 \\ x^2-x & \text{for } x \geq 4 \end{cases}$

(a) Determine $f(4)$ $f(4) = (4)^2 - 4 = 12$

(b) Find $\lim_{x \rightarrow 4^+} f(x)$ $\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} (x^2 - x) = 12$

(c) Complete the definition (using limits): f is **continuous** at $x = a$ if

$$\lim_{x \rightarrow a} (f(x)) = f(a)$$

(d) Using the definition in (c), explain carefully why $f(x)$ is not continuous at $x = 4$.

note that $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} (8-x) = 4 \neq \lim_{x \rightarrow 4^+} f(x)$

so the limit as $x \rightarrow 4$ of $f(x)$ does not exist.

(5 pts) The depth of water at a particular point in a bay is given by the function $y = 20 + 8 \cos(t/2)$, where y is in feet and t is time in hours since midnight.

(a) What is the amplitude?

$$\text{amplitude} = 8$$

(b) Find the period for this function..

$$4\pi: \frac{2\pi}{\text{period}} = \frac{1}{2} \Rightarrow \text{period} = 4\pi$$

(c) What is the depth of the water at low tide?

$$\text{depth @ low tide} = 20 - 8 = 12$$

(d) Give one time when tide occurs.

$$\text{high tide: } t = 0, t = 4\pi, t = 8\pi \text{ etc.}$$

$$\text{low tide: } t = 2\pi, t = 6\pi, \text{ etc.}$$