Name:______________________________ Section:__________

Directions: Please read each question carefully. Solution methods must be complete, logical and understandable, answers must be clearly labeled and explanations must be clearly written in the space provided. Calculators are allowed but you must show all your work to receive full credit on a problem.

1. (10 pts)

(a) State the formal definition of $\lim_{x\to c} f(x) = L$.

$\lim_{x\to c} f(x) = L$ means: for every $\varepsilon > 0$ there exists $\delta > 0$ such that $0 < |x-c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

(b) Briefly explain the meaning of your definition in practical terms:

this definition means that if one wants $f(x)$ close to L (with some given error), one can find a small interval of width 2δ centered at c which guarantees closeness

2. (10 pts)

(a) State the formal definition of $f'(a)$, the *derivative* of f at the point $x = a$.

$$f'(a) = \lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a}\right] = \lim_{h\to 0}\left[\frac{f(a+h)-f(a)}{h}\right]$$

(b) Use your definition in (a) to calculate $f'(2)$, where $f(x) = 4 - x^2$. You receive no credit for using a rule.

$$f'(2) = \lim_{h\to 0}\left[\frac{1}{h}\left(f(2+h) - f(2)\right)\right] = \lim_{h\to 0}\left[\frac{1}{h}\left(4-(2+h)^2 - 0\right)\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{h}\left(\cancel{4} - \cancel{4} - 4h - h^2\right)\right] = \lim_{h\to 0}\left[\frac{1}{h}(-4h - h^2)\right] = -4$$