Practice Problems

Math 115

Dec. 6, 2004

These problems are not in any order. Also, I haven't seen the draft of the final exam yet, so this practice sheet is a little speculative.

1. Find the domain of definition of each of the following functions.
   (a) $f(x) = \frac{x-2}{x^2-9}$
   (b) $f(x) = \sqrt{1-x^2}$
   (c) $f(x) = \ln(x^2-4)$
   (d) $\tan(2x)$

2. Solve each equality (or inequality) below for x.
   (a) $|x-3| = 2$
   (b) $\frac{x^2-1}{x-2} = 4$
   (c) $\ln(x^2-2x) = 0$
   (d) $|x-4| \leq 3$

3. Evaluate each of the following limits. If the limit does not exist, be sure to explain why.
   (a) $\lim_{x\to 1} \frac{x^2-1}{x-1}$
   (b) $\lim_{x\to 0} \sin(1/x)$
   (c) $\lim_{x\to 0} x\sin(1/x)$
   (d) $\lim_{x\to\infty} \frac{x^3+2x^2-x+1}{2x^3-x^2+2}$
   (e) $\lim_{x\to\infty} \cos(x)$

4. Provide a $\delta - \epsilon$ proof for each of the following limits.
   (a) $\lim_{x\to 2}(3x-4) = 2$
   (b) $\lim_{x\to 1}(x^2+1) = 2$
   (c) $\lim_{x\to 8} x^{1/3} = 2$

5. For each of the given functions $f(x)$ below, decide whether it is continuous at the point listed. Be sure to justify your answer.
   (a) $f(x) = \begin{cases} x+1 & x \leq 0 \\ e^x & x > 0 \end{cases}$, $x_0 = 0$
   (b) $f(x) = |x-2|$, $x_0 = 2$
   (c) $f(x) = \begin{cases} \frac{x^2-1}{x-1} & x \neq 1 \\ 2 & x = 1 \end{cases}$, $x_0 = 1$
   (d) $f(x) = \begin{cases} \frac{x^2-1}{|x-1|} & x \neq 1 \\ 2 & x = 1 \end{cases}$, $x_0 = 1$

6. A friend of your claims that a function is continuous if and only if you can write it down as one formula. Do you agree or disagree?

7. Differentiate each of the following functions.
   (a) $f(x) = x\cos x$
   (b) $f(x) = \frac{x^2-4}{x^3+1}$
   (c) $\sqrt{x^2+1}$
   (d) $e^{\sin(x)}$
   (e) $\ln(1+x^2)$

8. Find the tangent line to each graph/curve at the specified point.
   (a) the graph $y = x\sqrt{2x+1}$, $x_0 = 0$
   (b) the graph $y = \tan x$, $x_0 = \pi/4$
   (c) the curve $4 = x^2 + 4y^2$, $(x_0, y_0) = (\sqrt{3}, 1/2)$
   (d) the curve $xy = 4$, $(x_0, y_0) = (-2, -2)$
   (e) the curve $ye^x = 1$, $(x_0, y_0) = (0, 1)$

9. Consider the function $f(x) = x^5 - 5x^3 + 10x$ for $-3 \le x \le 3$.
   (a) Find all the critical points of f on the interval $[-3, 3]$.
   (b) Classify these critical points as maxima, minima, or neither.
   (c) What is the maximum value of f on $[-3, 3]$? Where does f achieve its maximum?
   (d) Where in $[-3, 3]$ is f increasing?
   (e) Where in $[-3, 3]$ is f concave up?
   (f) Sketch a graph of f.

10. A friend of yours claims that all continuous functions are differentiable. Do you agree or disagree?

11. Suppose a photographer is covering the 100m dash at a track and field event. She positions her camera on a stand 50m from the starting line and 5m from the track. Assume the racer run 10m/s. If the photographer tracks the racersduring the race, how fast is the angle her camera makes with the track changing when the racers pass her?

12. Suppose a 5ft tall person is walking towards a 15ft lamp-post at a rate of 5ft/s. When he is 15 ft from the base of the lamp-post, how fast is the length of his shadow decreasing?

13. Evaluate the following limits.
   (a) $\lim_{x\to 0} \frac{e^x-1}{x^2-x}$
   (b) $\lim_{x\to 1} \frac{x-1}{\ln x}$
   (c) $\lim_{x\to 1} \frac{\ln x}{\sqrt{x}}$

14. Evaluate each of the following definite integrals.
   (a) $\int_1^2 (x^2 - 3x^3 + x)dx$
   (b) $\int_0^\pi \cos x dx$
   (c) $\int_0^1 x\sqrt{1+x^2}dx$
   (d) $\int_0^{\pi/4} \tan x dx$
   (e) $\int_0^1 xe^{x^2}dx$

15. Write each of the areas described below as a definite integral. You do not need to evaluate the integral.
   (a) the area between $y = 0$ and $y = \sin x$, for $0 \le x \le \pi$
   (b) the area between $y = 0$ and $y = \cos x$ for $0 \le x \le \pi$
   (c) the area between $y = x$ and $y = x^3$ for $0 \le x \le 1$

16. Explain the inequality $|\int_a^b f(x)dx| \le \int_a^b |f(x)|dx$ in terms of areas. When are the two equal?

17. Evaluate each of the following derivatives.

    (a) $\frac{d}{dx}\int_0^x t^2 dt$

    (b) $\frac{d}{dx}\int_1^x t^2 dt$

    (c) $\frac{d}{dx}\int_x^1 t^3 dt$

    (d) $\frac{d}{dx}\int_0^{x^2} \sin t dt$

    (e) $\frac{d}{dx}\int_{\sin x}^x e^t dt$

    (f) $\frac{d}{dx}\int_{x^2-3e^x}^{x^2-3e^x} t^{3/2}\sin t dt$

18. Find the volumes described below.

    (a) the volume formed by rotating $y \leq x^2$ about the x-axis for $0 \leq x \leq 1$

    (b) the volume formed by rotating $x \leq \sin y$ about the y-axis for $0 \leq y \leq \pi$

19. Find the length of the curves described below.

    (a) the graph of $y = x^2 + 1$ for $0 \leq x \leq 1$

    (b) the curve given by $x(t) = 2\cos t, y(t) = \sin t$ for $0 \leq t \leq \pi$