Practice Problems

Math 115

Oct. 28 2004

These problems are not in any particular order. The exam will be shorter (about 4 or 5 problems).

1. Differentiate each of the following functions.
   (a) $f(x) = x\cos(x)$
   (b) $f(x) = \frac{e^x}{1+x^2}$
   (c) $f(x) = x\ln x$
   (d) $f(x) = e^{\sin(x)}$
   (e) $f(x) = \frac{\cos(x^2)}{e^x}$
   (f) $f(x) = \sqrt{1+x^2}$.
2. Find the tangent line to each graph/curve at the specified point.
   (a) $f(x) = xe^x$, $x_0 = 1$
   (b) $f(x) = \tan x$, $x_0 = \pi/4$
   (c) $f(x) = \frac{x-2}{x^2+1}$, $x_0 = -1$
   (d) $x^2 - y^3 = 0$, $(x_0, y_0) = (1, 1)$
   (e) $y\cos x + x\sin y = 0$, $(x_0, y_0) = (0, \pi)$
3. For each of the functions below, compute the derivative by evaluating the limit
$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$
   (a) $f(x) = 2x^3 - 3x^2 + 1$
   (b) $f(x) = (x+1)^2$
4. Consider the function $f(x) = x^3 - 2x^2 + 1$ for $-2 \le x \le 2$.
   (a) What are the critical points of f in the interval $[-2, 2]$?
   (b) Characterize these critical points as local maxima or local minima as well as you can.
   (c) What are the inflection points of f in the interval $[-2, 2]$?
   (d) What is the maximum of f on $[-2, 2]$, and all the values of x where f assumes its maximum. Do the same thing for the minimum.
   (e) Sketch the graph of f.
5. Suppose a 6 foot tall person is walking away from an 18 foot tall lamp-post at a rate of 2 ft/s. When the person is 12 feet from the lamp-post, how fast is her shadow lengthening?
6. Suppose you're making an open-top box from a rectangular sheet of cardboard, where the sheet of cardboard is 50 cm by 70 cm. You're going to make this box by cutting squares of side length x from each corner and folding the flaps up. (The squares all have to have the same side length, or else the flaps won't match up.) What value of x maximizes the volume of the box?
7. Suppose you're pouring water into a cylindrical tank of radius 5 m, at a rate of 10 m^3/s. When the total volume of water is 30π, how fast is the water level rising?
8. Let f and g be differentiable functions and suppose $f - g$ is a strictly increasing function.
   (a) If $f'(x_0) = 0$, is g increasing or decreasing near x_0?
   (b) If f is increasing near x_0, is $g'(x_0)$ positive, negative, zero, or undetermined?