Practice Problems

Math 115

Sept. 17, 2004

These problems are not in any particular order. The exam will be shorter (about 4 or 5 problems).

1. Find the domain of definition of the following functions:
   (a) $\sqrt{1-x^2}$
   (b) $\frac{x-2}{x^2-4}$
   (c) $|x-3|$
   (d) $\frac{x^2-9}{|x-3|}$
   (e) $\ln(x^2-1)$
   (f) $h(x) = f \circ g(x)$, where $g(x) = \sqrt{1-x^2}$ and $f(y) = y^2$

2. Solve for x in each of the following expressions. If the expression is an inequality, find the range of x for which the inequality holds.
   (a) $x = \sqrt{1-x}$
   (b) $e^x = 1$
   (c) $|x-1| < 1$
   (d) $|x+1| < x^2$
   (e) $\frac{1}{x+1} = x$

3. Determine whether each of the limits below exists. Compute the limits that do exist. Be sure to explain your answers!
   (a) $\lim_{x\to 0} \frac{x}{x^2-x}$
   (b) $\lim_{x\to 0} \tan(1/x)$
   (c) $\lim_{x\to 0} x\tan(1/x)$
   (d) $\lim_{x\to 2} \frac{x-2}{x^2-4}$
   (e) $\lim_{x\to 2} \frac{|x-2|}{x^2-4}$
   (f) $\lim_{x\to 0} \frac{\tan(x)}{\sin(x)}$
   (g) $\lim_{x\to\infty} e^{-x}$
   (h) $\lim_{x\to\infty} e^{x}$
   (i) $\lim_{x\to\infty} e^{\sin(x)}$
   (j) $\lim_{x\to\infty} \sin(e^{-x})$
   (k) $\lim_{x\to\infty} \frac{x^2}{x+1}$
   (l) $\lim_{x\to\infty} \frac{x}{x^2+1}$
   (m) $\lim_{x\to\infty} \frac{x}{x+1}$

4. Provide a δ-ϵ proof which justifies each of the following limit computations.
   (a) $\lim_{x\to 2} x^3 = 8$
   (b) $\lim_{x\to -2} x^2 = 3$
   (c) $\lim_{x\to 1} \frac{|x-1|}{x-1}$ DNE
   (d) $\lim_{x\to 0} \sqrt{x} = 0$

5. Decide whether each of the following functions is continuous at the given point. Be sure to explain your answers!

   (a) $x = 1,\ f(x) = |x - 1|$

   (b) $x = 0,\ f(x) = x^{1/4}$

   (c) $x = 2,\ f(x) = \begin{cases} \frac{x-2}{x^2-4} & x \neq 2 \\ 1/4 & x = 2 \end{cases}$

   (d) $x = 2,\ f(x) = \begin{cases} \frac{|x-2|}{x^2-4} & x \neq 2 \\ 1/4 & x = 2 \end{cases}$

   (e) $x = 0,\ f(x) = \begin{cases} \tan(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$

   (f) $x = 0,\ f(x) = \begin{cases} x\tan(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$

6. Prove that if two functions f and g are continuous at $x = a$ then the function $f + g$ is also continuous at $x = a$.

7. Find two functions $f(x)$ and $g(x)$ such that $f(0) = 0 = g(0)$ and

   (a) $\lim_{x\to 0} \frac{f(x)}{g(x)}$ DNE

   (b) $\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$

   (c) $\lim_{x\to 0} \frac{f(x)}{g(x)} = 2$

8. Suppose $f(x)$ is a function defined for $0 \leq xleq 1$, such that $0 \leq f(x) \leq 1$. For each of the statements below, either argue why it is correct or find a counter-example.

   (a) If f has an inverse function (i.e. there is a $g = g(y)$ such that $g(f(x)) = x$) then f is continuous.

   (b) If f is continuous then it has an inverse function.

   (c) If f is continuous and it has an inverse function then the inverse function is also continuous.