

Selected Solutions for Homework 6

Math 115

Oct. 19, 2004

1. (2.3.4)

We want to expand $(x + h)^n$, where n is a positive integer. Thus we have

$$(x + h)^n = (x + h)(x + h) \cdots (x + h).$$

Multiplying successive terms, we have one term of the form x^n , which occurs when all the factors of x are multiplied together in each parenthetical term. Also, there are n terms which multiply $n - 1$ factors of x together with one factor of h (in some order), so the coefficient on the $x^{n-1}h$ term is n . In general, the coefficient on the $x^k h^{n-k}$ is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

2. (2.3.12)

$$f(s) = 5\sqrt{s} - 4s^2 + 3 = 5s^{1/2} - 4s^2 + 3,$$

so

$$f' = \frac{5}{2}s^{-1/2} - 8s = \frac{5}{2\sqrt{s}} - 8s.$$

3. (2.3.25)

$$f'' = (x^4 + 3x^2 - 2)'' = (4x^3 + 6x)' = 12x^2 - 6$$

4. (2.3.28)

$$(2x^4 - 3x^{-1/2})'' = (8x^3 + (3/2)x^{-3/2})' = 24x^2 - \frac{9}{4}x^{-5/2}.$$

5. (2.3.44)

We want to find the tangent line to the graph of $f(x) = x^2 - 2x + 1$ at $x = 2$. First we compute the derivative:

$$f' = 2x - 2, \quad f'(2) = 2 \cdot 2 - 2 = 2.$$

So the slope of the tangent line is $f'(2) = 2$. The line also passes through the point $(2, f(2)) = (2, 1)$. So the equation of the line is

$$y - 1 = 2(x - 2).$$

6. (2.4.6)

$$f' = [(x^2 + 3)(x^3 - 3x + 1)]' = (x^2 + 3)'(x^3 - 3x + 1) + (x^2 + 3)(x^3 - 3x + 1)' = (2x)(x^3 - 3x + 1) + (x^2 + 3)(3x^2 - 3).$$

7. (2.4.12)

$$f' = \left[\frac{2x + 5}{5x - 1} \right]' = \frac{(2x + 5)'(5x - 1) - (2x + 5)(5x - 1)'}{(5x - 1)^2} = \frac{2(5x - 1) - 5(2x + 5)}{(5x - 1)^2} = -\frac{27}{(5x - 1)^2}.$$

8. (2.4.25)

For three terms in the product, we have

$$(fgh)' = ((fg)h)' = (fg)'h + (fg)h' = (f'g + fg')h + fgh' = f'gh + fg'h + fgh'.$$

In general, we will have n terms in the derivative of the product of f_1 through f_n . Each term will have the derivative of one of the f_j 's.

9. (2.4.37)

We know that the quantity is decreasing at a rate of 4%. This means in one unit of time, the number of units will decrease by 4% of the current number of units. So if the number of units at time t is $Q(t)$, then this number will decrease at a rate of $.04Q(t)$. So $Q'(t) = -.04Q(t)$ (the negative sign tells us the number of units decreases). Similar reasoning implies that if the price is increasing at 3% then $P' = .03P$. Now we use the product rule on the revenue $R = PQ$.

$$R' = (PQ)' = P'Q + PQ' = .03PQ - .04PQ = (.03 - .04)PQ = -.01PQ = -.01R.$$

Thus the revenue decreases at a rate of 1%.

10. (2.5.1)

The graph of $\sin x$ should have the same steepness as the line $y = x$ at the point $(0, 0)$. In fact, the steepness never is more than 1 or less than -1 . In other words, the angle the tangent line to the graph makes with the horizontal axis varies between $\pi/4$ and $-\pi/4$ (inclusive). This is because the derivative function $\cos x$ is bounded between 1 and -1 .

11. (2.5.6)

$$\frac{d}{dx}(x^2 + 2 \cos x) = (x^2)' + (2 \cos x)' = 2x - 2 \sin x$$

12. (2.5.14)

$$\frac{d}{dx} \left(\frac{x^2}{\csc x} \right) = \frac{d}{dx}(x^2 \sin x) = (x^2)' \sin x + x^2(\sin x)' = 2x \sin x + x^2 \cos x$$

13. (2.5.30)

We want to find the tangent line to the graph of $f(x) = \tan(x)$ at $x = 0$. First take the derivative:

$$f' = \frac{d}{dx}(\tan(x)) = \frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) = \frac{(\sin x)' \cos x - (\cos x)' \sin x}{\cos^2 x} = \frac{\cos^2 x - (-\sin^2 x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}.$$

Evaluating this at $x = 0$, we find that the slope of the tangent line is $f'(0) = 1/\cos^2(0) = 1$. This line also goes through the point $(0, f(0)) = (0, 0)$, so it has the equation $y = x$.